Dear Editor:

Thank you very much for providing the opportunity for us to revise our paper.

Thank you very much for your contributions to this paper. And we are all extremely grateful for having a chance to make further improvements. Reading and considering all comments of two reviewers carefully, we have made major revisions on our paper. The major three suggestions of reviewer1 and detail comments on the 4 points of reviewer2 are very helpful for us. Following the two reviewers’ suggestions, we have made major revisions on our paper.

Finally, we write the point-by-point response to answer the two reviewers’ questions for better communication. If there are still any problems on the method, diction, phrasing, grammar, and spelling, please do not hesitate to tell us and we’ll try our best to improve them.

Thank you again for your comments to improve our paper. Wish your journal better and better.

Yours,

Mei Hong

2018-02-06
Responses to reviewer#1:

All the authors are extremely grateful to you for providing your excellent comments and valuable advices for this paper. Your major suggestions that the reliability of this datasets is not mentioned and the authors did not verify their results in spring season are very helpful for us. Based on your suggestions, we have made some revisions to on our paper. We have added the discussion of reliability of this datasets and the new results in spring season based on your specific comments.

Thank you again for your valuable comments to improve our submission. If there are still any problems on the method, diction, phrasing, grammar, and spelling, please do not hesitate to tell us and we’ll try our best to improve them.

In the following, kind comments you suggested before are in black text with corresponding actions taken by us following in blue.

Specific comments:

1. The method used in this study is based on the statistic regression, which basically depends on the quality of observations. In section 2.1, although the authors claimed that the monthly average SST data from the UK Met Office Hadley Centre is adopted in this study, the reliability of this datasets is not mentioned. Besides, the verification of this datasets with in-situ observation is also strongly recommended by this reviewer.

Responses: Good suggestions. In the previous paper, we have neglected the discussion of reliability of this datasets. Now there are three main categories of SST
data. The gridded 2°×2° NOAA Extended Reconstructed SST dataset (ERSST.v3b; Smith et al. 2008) includes in situ data (ships and buoys), but does not include satellite data. The gridded 1°×1° Met Office Hadley Sea Ice and SST dataset (HadISST1; Rayner et al. 2003) includes both in situ and available satellite data. The gridded 1°×1° NOAA Optimal Interpolation SST (OISST.v2; Reynolds et al. 2002) incorporates in situ and satellite data, but unlike the other two SST datasets, it is only available in the recent period from November 1981 to the present. Both HadISST1 and ERSST.v3b are available from the mid-to-late 1800s, but only monthly data from 1951 to 2010 was considered in this study.

Considering comprehensively, the gridded 1°×1° Met Office Hadley Sea Ice and SST dataset data, no matter from data quality or data length, is the most appropriate to used.

The specific revision can be seen from line118 to line120 in page6.

We sincerely hope for your satisfaction with our revision. Thank you again for your kind suggestion.

References:


doi:10.1029/2002JD002670

situ and satellite SST analysis for climate. J Clim 15:1609–1625

2. One important conclusion of this study is “The difference between forecast results
in summer and those in winter is not high, indicating that the improved model can
overcome the spring predictability barrier to some extent”. This conclusion is vague
and lack of rigorous verification because the authors did not verify their results in
spring season.

Responses: Good suggestions. The skill of forecasts that start in February or May
drops faster than that of forecasts that start in August or November. This behavior,
often termed the spring predictability barrier, is in part because predictions starting
from February or May contain more events in the decaying phase of ENSO (Jin et al.,
2008). Based on the reviewer’s suggestion, we have added the experiments in the
spring and in the autumn in Table4. From the table, we can see the forecast result in
spring of our model is also good, indicating that the improved model can overcome
the spring predictability barrier to some extent. The specific revision can be seen in
from page66.

We sincerely hope for your satisfaction with our revision. Thank you again for
your kind suggestion.

Table 4. Temporal correlation(TC) and the mean absolute percentage error (MAPE) between
model forecasts and observations within 12 months for Nov.–Jan., Dec.–Feb., and Jan.–Mar. as
lead time of winter, for Feb.–Apr. , Mar.–May and Apr.–June as lead time of spring, for May–July,
as lead time of autumn.

<table>
<thead>
<tr>
<th>Forecast events</th>
<th>Lead time of all seasons combined (MJJ-JJA-S)</th>
<th>Lead time of summer (ASO-SON-OND)</th>
<th>Lead time of autumn (NDJ-DJF-JFM)</th>
<th>Lead time of winter (FMA-MAM-AMJ)</th>
<th>Lead time of spring</th>
</tr>
</thead>
<tbody>
<tr>
<td>TC MAPE</td>
<td>TC MAPE</td>
<td>TC MAPE</td>
<td>TC MAPE</td>
<td>TC MAPE</td>
<td></td>
</tr>
<tr>
<td>The average of</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>18 El Niño</td>
<td>0.604 9.70%</td>
<td>0.569 10.33%</td>
<td>0.632 8.85%</td>
<td>0.677 8.02%</td>
<td>0.538 11.6%</td>
</tr>
<tr>
<td>examples</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>The average of</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>22 La Niña</td>
<td>0.625 8.97%</td>
<td>0.581 9.82%</td>
<td>0.645 8.41%</td>
<td>0.695 7.83%</td>
<td>0.579 9.82%</td>
</tr>
<tr>
<td>examples</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>The average of</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20 Neutral</td>
<td>0.798 5.96%</td>
<td>0.752 6.86%</td>
<td>0.831 5.31%</td>
<td>0.844 4.60%</td>
<td>0.765 7.07%</td>
</tr>
<tr>
<td>examples</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>The average of</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>total 60</td>
<td>0.712 7.62%</td>
<td>0.633 8.51%</td>
<td>0.786 6.88%</td>
<td>0.776 6.52%</td>
<td>0.653 8.03%</td>
</tr>
<tr>
<td>examples</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3. Lines 42-44. Compared with six mature models published previously, the present model has an advantage in prediction precision and length, and is a novel exploration of the ENSO forecast method”. The major concerns of this reviewer are: what is the sample size in comparing the forecast results? Are those samples really representative?

Responses: Good suggestions. As shown in Table 4, our ENSO forecast is a total of 60 experiments, including 18 El Niño examples, 22 La Niña examples, and 20 Neutral examples, and each experiment contains lead time of four seasons. Finally, it is the equivalent of 240 experiments. Figure 11 and Figure 12 is the average TC and RMSE of the 240 experiments of compared with six mature models, covers a variety of different types of ENSO and different lead time. So those samples should be really
representative. We haven't explained it in previous paper, and now we explain it from line 564 to 567 on page 27.

We sincerely hope for your satisfaction with our revision. Thank you again for your kind suggestion.

Minor comments:

1. Line 122, give the full name of “SOI”.

Responses: Good suggestions. Now we have given the full name of “SOI” as the Southern Oscillation Index (SOI) in line 128 in page 6.

We sincerely hope for your satisfaction with our revision. Thank you again for your kind suggestion.

2. Line 549, “mode” should be “model”.

Responses: Good suggestions. Now we have revised “mode” as “model” in line 545 in page 26.

We sincerely hope for your satisfaction with our revision. Thank you again for your kind suggestion.

Responses to reviewer #2:

All the authors are extremely grateful to you for providing your excellent comments and valuable advices for this paper. Your major four suggestions that Construction of the first two predictors ie $T_1$ and $T_2$; Selection of the other predictors; Structure of the model and Model validation are very helpful for us. Based on your
suggestions, we have made major revisions to our paper. We have added the
discussion of the selection of the predictors, the structure of the model and the model
validation based on your specific comments.

Thank you again for your valuable comments to improve our submission. If there
are still any problems on the method, diction, phrasing, grammar, and spelling, please
do not hesitate to tell us and we’ll try our best to improve them.

In the following, kind comments you suggested before are in black text with
corresponding actions taken by us following in blue.

1. Section 2.2 EOF deconstruction. This section requires some more detail. While
the given reference describes the EOF method, we need to know how it is applied
here. Is the correlation or covariance matrix used? How are the anomalies constructed
– simple removal of the monthly means? How are the anomalies smoothed - how
strong is the smoothing and is it applied spatially or over time? More importantly,
why are only the first 2 EOFs considered? A similar analysis has recently been
Their first two EOFs are similar to those described here (but with no smoothing and
hence lower explained variance). Using different data sets and time periods, they
show that the 2nd EOF is not stable, being entirely due to the strong trend (also
evident in Figure 1d). The pattern does not appear if the data is detrended, and also
becomes less important if different time periods and/or domains are used. Most
importantly, they do not interpret it as indicating “the ENSO signal beginning to
decay”.
Responses: Good suggestions. We have used covariance matrix, because the covariance matrix was selected to diagnose the primary patterns of co-variability in the basin-wide SSTs, rather than the patterns of normalized covariance (or correlation matrix). We have used the smooths function with MATLAB, which is five points two times moving, mainly filtering out some noise points and outliers.

Because the variance contribution of the first EOF mode is 61.33% and the variance contribution of the second EOF mode is 14.52%, so the first two EOF modes account for 75.85% of the total variance contribution, which has occupied most of the variance contribution and also contains most of the information of the field decomposition. So the first 2 EOFs are considered.

Based on the reference of L'Heureux et al. (Clim Dyn 2013, DOI 10.1007/s00382-012-1331-2), we need to do more experiments to prove that we choose the second mode of EOF to be appropriate, and whether different time periods will make us forecast unstable or not. Our original data is the monthly average SST data from January 1951 to Dec. 2010, which are 60 years. We will increase the length of the data for 20 years (Jan.1931 –Dec.2010), for 10 years (Jan.1941- Dec.2010) and decrease the length of the data for 10 years (Jan.1961- Dec.2010), for 20 years (Jan.1971- Dec.2010). And then we use the same method to reconstruct a model and forecast the ENSO index as section5.4. The prediction results are shown in the following table:

Table 5. The forecast results of the different data periods

<table>
<thead>
<tr>
<th>Forecast events</th>
<th>The data periods (Jan.)</th>
<th>The data periods (Jan.)</th>
<th>The data periods (Jan.)</th>
<th>The data periods (Jan.)</th>
<th>The data periods (Jan.)</th>
<th>The data periods (Jan.)</th>
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<tbody>
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<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lead time of all seasons combined</td>
<td>Lead time of all seasons combined</td>
<td>Lead time of all seasons combined</td>
<td>Lead time of all seasons combined</td>
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</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TC</th>
<th>MAP E</th>
<th>TC</th>
<th>MAPE</th>
<th>TC</th>
<th>MAPE</th>
<th>TC</th>
<th>MAP E</th>
<th>TC</th>
<th>MAPE</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.60</td>
<td>0.70%</td>
<td>0.68</td>
<td>9.02%</td>
<td>0.642</td>
<td>9.35%</td>
<td>0.57</td>
<td>10.15%</td>
<td>0.551</td>
<td>10.44%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Lead time of all seasons combined</th>
<th>Lead time of all seasons combined</th>
<th>Lead time of all seasons combined</th>
<th>Lead time of all seasons combined</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>TC</th>
<th>MAP E</th>
<th>TC</th>
<th>MAPE</th>
<th>TC</th>
<th>MAPE</th>
<th>TC</th>
<th>MAP E</th>
<th>TC</th>
<th>MAPE</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.62</td>
<td>8.97%</td>
<td>0.70</td>
<td>8.33%</td>
<td>0.675</td>
<td>8.55%</td>
<td>0.58</td>
<td>9.42%</td>
<td>0.567</td>
<td>9.82%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Lead time of all seasons combined</th>
<th>Lead time of all seasons combined</th>
<th>Lead time of all seasons combined</th>
<th>Lead time of all seasons combined</th>
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<table>
<thead>
<tr>
<th>TC</th>
<th>MAP E</th>
<th>TC</th>
<th>MAPE</th>
<th>TC</th>
<th>MAPE</th>
<th>TC</th>
<th>MAP E</th>
<th>TC</th>
<th>MAPE</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.79</td>
<td>5.96%</td>
<td>0.84</td>
<td>5.12%</td>
<td>0.821</td>
<td>5.56%</td>
<td>0.74</td>
<td>6.21%</td>
<td>0.721</td>
<td>6.58%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Lead time of all seasons combined</th>
<th>Lead time of all seasons combined</th>
<th>Lead time of all seasons combined</th>
<th>Lead time of all seasons combined</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>TC</th>
<th>MAP E</th>
<th>TC</th>
<th>MAPE</th>
<th>TC</th>
<th>MAPE</th>
<th>TC</th>
<th>MAP E</th>
<th>TC</th>
<th>MAPE</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.71</td>
<td>7.62%</td>
<td>0.77</td>
<td>7.14%</td>
<td>0.740</td>
<td>7.38%</td>
<td>0.68</td>
<td>7.96%</td>
<td>0.652</td>
<td>8.15%</td>
</tr>
</tbody>
</table>

From the table, we can see that in the 60 experiments, the prediction results of the data period increased by 20 years are the best, and the prediction results of the data period decreased by 20 years is the worst. This is because the more data we use, the more information it contain. But from the table we can also see the difference among forecast results of both TC and MAPE of five different sample data are less, and no abnormal change suddenly worse or better appear. All these indicate that using different data sets and time periods, even though may have a certain impact on the pattern of the 2nd EOF, but the impact on our forecast is not great and it will not make our forecast unstable.

The "indicating the ENSO signal beginning to decay" in our previous paper is a mistake of writing, which is not seen from the space mode of Figure 1 (c), but from
We have added the discussion about the stability of our forecast in page6-7 and page28-29 and revised as "the ENSO signal beginning to enhanced " in page7.

We sincerely hope for your satisfaction with our revision. Thank you again for your kind suggestion.

2. Section 2.3 Predictor selection The selection of other potential predictors is confusing. Apart from T1 and T2, the other potential predictors come from a fairly limited set, and are not well supported by the referenced works. In lines 157-160, zonal winds in the western and eastern equatorial Pacific are mentioned, and it is well known that westerly wind anomalies in the western equatorial Pacific can (and do) trigger equatorially trapped oceanic Kelvin waves. There is an extensive amount of literature on the relationship between western equatorial Pacific zonal wind and ENSO, but here no references are given and only the eastern equatorial winds is considered. Trenberth et al. discuss a link between ENSO and the PNA pattern (amongst other modes of extratropical variability), but this is the context of ENSO forcing of the PNA, ie ENSO leads to PNA teleconnections, but PNA does not predict ENSO. Yang et al introduce the EAWM index, but they note that "the relationship between ENSO and the east Asian winter monsoon is relatively weak". Nowhere do
they suggest that the EAWMI is closely related to any ENSO indices. It is not surprising that the east Pacific wind and PNA do not feature in the final model.

Responses: Good suggestions. Your opinion is very good. In previous paper the factors that we may consider are relatively few. But we are a complex coupled model of four factor differential equations and are not the similar with a simple statistical model (such as stepwise regression). So in our previous paper using the stepwise regression method to select factors also has a problem. According to your opinion, we have read more literatures. We have expanded the scope of factor selection and revised the criterion of selecting factors, and the paragraph has revised as follows:

Considering the complexity of computation, the amount of variables in the equations of our model can’t be too large, usually 3 or 4 for the best. This has been explained in our previous studies (Zhang et al., 2006; Zhang et al., 2008). If there are more than 4 variables in the modeling equation, it will cause the amount of parameters such as $a_1, a_2, \ldots, b_1, b_2, \ldots$ too large. The huge computation makes it difficult to be precisely modeled. Thus, the total number of parameters in the model of five variables was 102, which may cause an overfitting problem. Hence, when we selected the model of five or six variables which entailed large amounts of computation that made precision difficult, and too many parameters might cause an overfitting phenomenon. If we choose only two or even fewer variables, the forecast performance is poor too. Too few variables cause too small reconstructed parameters, resulting in amounts of important information missing out in the model. Thus, four
variables are best for dynamically and accurately modeling. Because we have chosen
two time series in section 2.2 as the modeling objects, now we should select the other
two ENSO intensity impact factors.

The ENSO intensity impact factor is an important issue in ENSO prediction.

Previous studies have been completed in this area, which found that teleconnection
patterns, temperature, precipitation, wind and SSH may affect ENSO strength. For
example, Trenberth et al. (1998) noted that PNA, SOI and OLR in the Pacific
Intertropical Convergence Zone (ITCZ) are all closely related to ENSO.

Webster (1999) pointed out after the 1970, Indian Ocean dipole (IOD) is not only
affected by ENSO, but also affected the strength of ENSO (Ashok et al., 2001). Yoon
and Yeh (2010) reported that the Pacific Decadal Oscillation (PDO) disrupts the
linkage between El Niño and the following Northeast Asian summer monsoon
(NEASM) through inducing the Eurasian pattern in the mid-high latitudes. The vast
majority of studies (Tomita and Yasunari, 1996; Zhou and Wu, 2010; Kim et al.,
2017) have concentrated on the impacts of ENSO on the East Asian winter
monsoon (EAWM). During the EAWM season, ENSO generally reaches its mature
phase and has the most prominent impact on the climate. Wang et al. (1999a) and
Wang et al. (1999b) suggested that the zonal wind factors in the eastern and western
equatorial Pacific play a critical role in the phase of transition of the ENSO cycle,
which could excite eastward propagating Kelvin waves and affect the SSTA in the
equatorial Pacific. Zhao et al. (2012) analyzed the characteristics of the tropical
Pacific SSH field and its impact on ENSO events.
Based on the above analysis, we have selected nine factors, which may be closely related with the ENSO index (Niño3.4).

(1) The zonal wind in the eastern equatorial Pacific factor (u1) was calculated as the grid-point average of zonal wind in the area [5° S ~ 5° N, 150° W ~ 90° W].

(2) The zonal wind in the western equatorial Pacific factor (u2) was calculated as the grid-point average of zonal wind in the area [0° ~ 10° N; 135° E ~ 180° E].

(3) The PNA teleconnection factor was obtained from the CPC.

(4) The dipole mode index factor (DMI) was obtained from SSTA for June-July-August (JJA) based on Saji (1999) method.

(5) The SOI factor was obtained from the CPC.

(6) The PDOI factor was obtained from department of Atmospheric Sciences in the university of Washington. The web is http://tao.atmos.washington.edu/pdo/RDO.latest.

(7) The EAWM index (EAWMI) factor was proposed by Yang et al. (2002), which is defined by the meridional 850-hPa winds averaged over the region (20° ~40°N, 100°~140°E).

(8) The OLR in the ITCZ factor was calculated as the grid-point average of OLR in the area [10°N~20°N, 120°E~150°E].

(9) The SSH factor was calculated as the grid-point average of the SSH data in the area [10° S ~ 10° N; 120° E ~ 60° W].

A correlation analysis of the above factors was carried out and the results are shown in Table 2.
Table 2 shows that SOI and EAWMI have the stronger correlation with the front two time series $T_1, T_2$ than the other 7 factors. The results are also consistent with previous research (Clarke and Van Gorder, 2003; Drosdowsky, 2006; Zhang et al., 1996; Wang et al., 2008; Yang and Lu, 2014). Therefore, the first time series $T_1$, the second time series $T_2$, SOI and EAWMI will be selected as prediction model factors.

Table 2. The correlation analysis between the front two time series $T_1, T_2$ and nine impact factors

<table>
<thead>
<tr>
<th>factors</th>
<th>$u_1$</th>
<th>$u_2$</th>
<th>PNA</th>
<th>DMI</th>
<th>SOI</th>
<th>PDOI</th>
<th>EAWMI</th>
<th>OLR</th>
<th>SSH</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_1$</td>
<td>0.3161</td>
<td>0.5684</td>
<td>0.4386</td>
<td>-0.3457</td>
<td>0.7734</td>
<td>0.4081</td>
<td>0.6284</td>
<td>0.3287</td>
<td>0.3363</td>
</tr>
<tr>
<td>$T_2$</td>
<td>0.2118</td>
<td>0.4181</td>
<td>0.2560</td>
<td>-0.2345</td>
<td>0.5232</td>
<td>0.3065</td>
<td>0.4825</td>
<td>0.1816</td>
<td>0.2169</td>
</tr>
</tbody>
</table>

Actually, how many variables and which variables are used in our model become a key issue to be resolved. We are a complex four factor differential equations coupling model. We are a complex coupled model of four factor differential equations, so we are more concerned with the correlation between each other. The correlation must be considered as an important criterion to select the factors, but in order to further verify the correctness of the selection criterion, we have carried out the prediction experiments (the 60 cross-validated retroactive hindcasts experiments of the ENSO index for all seasons combined at lead times of 8 months) of different variables. The forecast results of the models of different variables are as following:

Table 3. The forecast results (The temporal correlation (TC) and the root mean square error (RMSE)) of the models of different variables
<table>
<thead>
<tr>
<th>Results</th>
<th>$T_1, T_2, u_1$</th>
<th>$T_1, T_2, u_2$</th>
<th>$T_1, T_2, PNA$</th>
<th>$T_1, T_2, DMI$</th>
<th>$T_1, T_2, SOI$</th>
<th>$T_1, T_2, PDOI$</th>
</tr>
</thead>
<tbody>
<tr>
<td>TC</td>
<td>0.4423</td>
<td>0.5628</td>
<td>0.3852</td>
<td>0.3226</td>
<td>0.6027</td>
<td>0.3809</td>
</tr>
<tr>
<td>RMSE</td>
<td>0.9025</td>
<td>0.7855</td>
<td>0.9244</td>
<td>1.0041</td>
<td>0.7275</td>
<td>1.0642</td>
</tr>
<tr>
<td>$T_1, T_2, EAWMI$</td>
<td>$T_1, T_2, OL R$</td>
<td>$T_1, T_2, SSH$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TC</td>
<td>0.5829</td>
<td>0.3205</td>
<td>0.4288</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RMSE</td>
<td>0.7516</td>
<td>0.9814</td>
<td>0.9090</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Four variables of the model

<table>
<thead>
<tr>
<th>Results</th>
<th>$T_1, T_2, u_1, u_2$</th>
<th>$T_1, T_2, u_1, PNA$</th>
<th>$T_1, T_2, u_1, DMI$</th>
<th>$T_1, T_2, u_1, SOI$</th>
<th>$T_1, T_2, u_1, PDOI$</th>
<th>$T_1, T_2, u_1, EAWMI$</th>
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<tbody>
<tr>
<td>TC</td>
<td>0.4672</td>
<td>0.3628</td>
<td>0.5617</td>
<td>0.5201</td>
<td>0.5028</td>
<td>0.5822</td>
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<td>0.9902</td>
<td>0.7617</td>
<td>0.8233</td>
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<td>$T_1, T_2, u_1, SSH$</td>
<td>$T_1, T_2, u_1, PNA$</td>
<td>$T_1, T_2, u_1, DMI$</td>
<td>$T_1, T_2, u_1, SOI$</td>
<td>$T_1, T_2, u_1, PDOI$</td>
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<tr>
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<td>0.4128</td>
<td>0.3107</td>
<td>0.4125</td>
<td>0.5910</td>
<td>0.5504</td>
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<tr>
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<td>0.9017</td>
<td>1.0255</td>
<td>0.9392</td>
<td>0.7128</td>
<td>0.7503</td>
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<td>$T_1, T_2, u_1, SSH$</td>
<td>$T_1, T_2, u_1, PNA$</td>
<td>$T_1, T_2, u_1, DMI$</td>
<td>$T_1, T_2, u_1, SOI$</td>
<td>$T_1, T_2, u_1, PDOI$</td>
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<tr>
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<td>0.6048</td>
<td>0.4528</td>
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<td>$T_1, T_2, PNA, DMI$</td>
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<td>$T_1, T_2, PNA, PDOI$</td>
<td></td>
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<tr>
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<tr>
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<td>0.7425</td>
<td>1.2905</td>
<td>0.7015</td>
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<tr>
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<td>$T_1, T_2, DMI, SSH$</td>
<td>$T_1, T_2, DMI, SOI$</td>
<td>$T_1, T_2, DMI, PDOI$</td>
<td>$T_1, T_2, DMI, EAWMI$</td>
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<tr>
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<td>0.6022</td>
<td>0.6344</td>
<td>0.5876</td>
<td>0.5476</td>
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</tbody>
</table>
From the table, we can see that for all the forecast results of the models of different variables, the prediction results of $T_j,T_k,SOI$ is the best among those of the three factors and the prediction result of $T_j,T_k,SOI,EAWMI$ is the best among those of the four factors. But the prediction result of $T_j,T_k,SOI,EAWMI$ is best among all, which proves that our selection factors are correct. In our previous study (Hong et al., 2015), the model of the Western Pacific subtropical high was established by using the correlations as a criterion to select factors and their forecast results are also good. Now we use the correlations as a criterion to select factors is also in line with our previous research.

With the deepening of the research, there are still a lot of new literatures that reveal the relationship between ENSO and the East Asian winter monsoon. For example:


So there is a good correlation between ENSO and the East Asian winter monsoon. The specific revision can be seen in section 2.3 in page 7-10 and line 616 to 632 in page 29-30. We sincerely hope for your satisfaction with our revision. Thank you again for your kind suggestion.

References:


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354 3-1. The remainder of section 2.3, concerned with determining the number of predictors is difficult to follow. It is not until section 3 (page 11) that it is revealed that the model is a dynamical system of four second order coupled equations, involving the products of the various predictors as well as the predictors themselves. Nowhere is the inclusion of these terms discussed or justified. What physical processes do these terms represent? What do the predictors squared represent?, and the cross products ie what do T1 * SOI or T2 * EAWMI mean? Since the model is not a linear regression model, is stepwise regression a valid procedure for determining the significance of the predictors?

Responses: Good suggestions. Your opinion is very good. Based on your suggestion of question 2, we have revised the discussion of how to determine the number of predictors. Our model is not a linear regression model, the stepwise regression may be a valid procedure for determining the significance of the predictors,
so we also have revised the method for determining the significance of the predictors, the specific revision can be seen our answer of the question.

The inclusion of these terms and the physical processes do these terms represent are important, especially for the discussion of dynamical characteristics of the dynamical model. But now we are difficult to give a clear meaning. Now the main work of our paper is the prediction experiments of the model. For the reason of time and length, this paper mainly discusses the prediction results of the model. The physical processes do these terms represent and the discussion of the dynamical characteristics of the model will be the focus of our next work. Before this, we have also used the Takens’ delay embedding theorem to reconstruct the dynamical model of the Western Pacific subtropical high (WPSH). And Based on the reconstructed dynamical model, dynamical characteristics of WPSH are analyzed and an aberrance mechanism is developed, in which the external forcings resulting in the WPSH anomalies are explored, which have been published (Hong et al., 2016). We also study the bifurcation and catastrophe of the West Pacific subtropical high ridge index of a nonlinear model (Hong et al., 2017). Based on our previous method and work, our next work is to analyze the physical processes and the dynamical characteristics of the SST field.

The specific revision can be seen from line 689 to 704 in page 33. We sincerely hope for your satisfaction with our revision. Thank you again for your kind suggestion.

References:


3-2. line 195. The idea that a model with the number of predictors less than 10% of the sample size can avoid overfitting is new to me. The reference given (Tetko et al) is about neural networks. Is this applicable to the system of coupled equations used here? (I could only see the first page) Also I am not sure if the discussion in 198-203 is incorrect. Even if only 34 parameters are accepted, the full set of 56 parameters must be estimated to know which to accept or reject. This may be more a problem of introducing artificial skill, which has long been recognised as a problem in statistical forecasting. It generally arises when you try enough predictors, and retain those that "work" and discard the others.

This question of the number of parameters / predictors is exacerabated in Section 4 and 5 where the number of predictors is increased again by including lagged values. On first inspection Equations 3 and 7 involve 112 parameters. There are 28 alphas, 28 thetas, as given in lines 395 and 396. (In line 202, it is stated that there are 28 self memorization parameters beta; but there are no betas in Eqs 3 and 5, but there are in Appendix B) In addition each of the four F "dynamical cores" involve 14 parameters
as shown in Equation 1, assuming that the same F is used at each lagged time. Given
that the input data (the xi) are different at each lag, is the same F a valid assumption?

Even with the authors 34 accepted values in the Fs, there is still a total of 90
parameters. This is well over 10%, and on the authors own criterion, this would
suggest that the system is perhaps overfit. Additionally, all the 720 observations are
not statistically independent. Both T1 and the SOI (and probably T2 with its strong
trend) are strongly auto-correlated, and the effective sample size is probably
significantly less than 720. All in all, this discussion is very confusing!

Responses: Good suggestions. Our final number of 90 parameters is still a little
large for a sample size of 720. In the previous paper, this discussion of overfitting is a
little confusing. So it is still necessary to further discuss whether our model has the
overfitting problem or not. Thank reviewers to remind us this problem.

The definition of overfitting: The learned hypothesis may fit the training set very
well, but fail to predict to new examples (fail to fit additional data or predict future
observations reliably).

The potential for overfitting depends not only on the number of parameters and
data but also the conformability of the model structure with the data shape, and the
magnitude of model error compared to the expected level of noise or error in the
data (Burnham and Anderson, 2002). So there are many reasons causing the overfitting
phenomenon. But this does not mean having many parameters relative to the number
of observations inevitably causes the overfitting problem (Golbraikh et al., 2003).
There is no evidence that more parameters will be certain to result in overfitting.
Based on the definition of overfitting and the previous studies (Golbraikh et al., 2003; Everitt and Skrondal, 2010), we can judge whether a model is overfitting or not by the accuracy of prediction results of independent samples (Golbraikh and Tropsha, 2002; Qi and Li, 2006).

In the sample training, our model does not purposely pursue the high degree of the training samples fitting and improve the effectiveness of the independent generalization. In fact in our paper the forecast results of the Cross-validated retroactive hindcasts (section 5.2) and the independent samples validation (table 3 and table 4) are both good. Especially, the independent samples validation of the ENSO index as the table 4, we have carried out the 240 independent sample validation prediction of four seasons of different ENSO events and the coverage of independent samples test is very wide. Moreover, compared with 6 mature prediction models, the forecast results of our model are also good, which prove the overfitting problem does not exist in our model. According to the previous literature (Islam and Sivakumar, 2002; Sivakumar et al., 2001), we can see that prediction principle and structure of the phase space reconstruction (PSR) of dynamical system is not the same with the traditional neural network and in the small sample situation the forecasting results of PSR model are better than those of the traditional neural network (Sivakumar et al., 2002), which can be verified in the independent sample test (table 3 and table 4). So according to the definition of overfitting, we can say the overfitting phenomenon does not exist in our model.
Now we have added the new discussion of the overfitting problem from line 63 to 66 in page 30-31.

We sincerely hope for your satisfaction with our revision. Thank you again for your kind suggestion.

References:


4. Model Validation

This paragraph took me a long time to understand, especially how one could obtain correlations and MAPE values based on a single forecast. As I understand it, "at this time" refers to the forecast at five months, and the correlation and MAPE are calculated over the first five months forecasts, and in general the values at the Nth month are based on the first N months forecast. (I assume that this is the "n" in the equation for MAPE on line 283)

Responses: Good suggestions. Your understanding is right. "at this time" refers to the forecast at five months, and the correlation and MAPE are calculated over the first five months forecasts, and in general the values at the Nth month are based on the first N months forecast. Now we revise the sentence "Using $\tau_i$ as an example, at this time, the temporal correlation between model predictions and corresponding observations was 0.8966 and the mean absolute percentage error (MAPE) (Hu et al., 2001), $\text{MAPE} = \frac{1}{n} \sum_{i=1}^{n} \left| \frac{D_p(i) - D_o(i)}{D_o(i)} \right| \times 100$, was 8.32%." as "Using $\tau_i$ as an example, the CC between model predictions and corresponding observations over the first five months forecasts was 0.8966 and MAPE was 8.32%." for readers’ better understanding.
The specific revision can be seen from line 275 to 276 in page 13. We sincerely hope for your satisfaction with our revision. Thank you again for your kind suggestion.

4-2. This method would suggest that the correlation at one month is undefined, and 1.0 (perfectly accurate) at two months? This same type of calculation appears to be used in Tables 3 and 4.

Responses: Good suggestions. In previous paper, we have not explained the concept of correlation. There are two different correlations in our paper. The first correlation in our paper is the Pearson correlation coefficient (CC), which also can be called the linear correlation coefficient. It measures the strength and direction of a linear relationship between two variables (for example model output and observed values).

The mathematical formula for computing $r$ is:

$$ r = \frac{\sum_{i=1}^{n} (D_x(i) - \bar{D}_x) \cdot (D_y(i) - \bar{D}_y)}{\sqrt{\sum_{i=1}^{n} (D_x(i) - \bar{D}_x)^2 \cdot \sum_{i=1}^{n} (D_y(i) - \bar{D}_y)^2}} $$

Where $n$ is the number of pairs of data, $D_x, D_y$ is a series of $n$ observations and $n$ forecast values.

The CC (Wang et al. 2009) and the mean absolute percentage error (MAPE) (Hu et al. 2001) are employed as objective functions to calibrate the model. The CC evaluates the linear relationship between the observed and predicting values and MAPE measures the difference between the observed and predicting values. The forecast results of $T_1, T_2$ in Section 3, table 2 and table 3 have used the above two evaluation criteria ($r$ and MAPE).
While the evaluation criteria of the ENSO index in table 4 is the temporal correlation (TC), its definition and specific calculation steps can be seen in these literatures (Kathrin et al., 2016; Nicosia et al., 2013). The TC is often used to measure the prediction effect of the ENSO index. For example, in 1995, Chen et al. used TC as the evaluation criteria to test the improved predictability of El Nino forecasting of their model and Barnston et al. in 2012 also used the TC to compare the forecast skill of 21 real-time seasonal ENSO models.

In the previous paper, we didn’t explain two different correlations clearly, which will be easy for readers to misunderstand. Now we have explained two different correlations and the specific revision can be seen in all my paper.

We sincerely hope for your satisfaction with our revision. Thank you again for your kind suggestion.

References:


4-3. line 289-298. Another confusing paragraph. January 1951 to January 1952 inclusive? is 13, not 12 months. How was the omitted section forecast, ie was it simply a 12 (or 13) month forecast starting at the last point before the omitted data?

Responses: Good suggestions. This is a mistake in writing and thanks the reviewers’ comments. The omitted forecast section is 12 months, Jan. 1951 to Dec. 1951, and the training sample time is Jan.1952 to December 2010. Then in the next prediction experiment, the omitted segment is Jan.1952 to Dec. 1952 and the training samples are Jan. 1951 to Dec.1951 and Jan.1953 to Dec.2010. So the forecast time series is Jan.1952 to Dec. 1952. We then repeated this procedure the by moving the omitted segment along the entirety of the available time series. The similar process of the cross-validated retroactive hindcasts has also been used in the previous literatures (Hu et al., 2017).

The specific revision can be seen from line 284 to 293 in page 14.

We sincerely hope for your satisfaction with our revision. Thank you again for your kind suggestion.

References:
Hu Y. J., Zhong Z., Zhu Y. M. et al., A statistical forecast model using the
time-scale decomposition technique to predict rainfall during flood period over the
middle and lower reaches of the Yangtze River Valley, Theoretical and Applied

it is difficult to judge how "good" the forecast was
based on Figure 3.

Responses: Good suggestions. From Fig3, the prediction values (blue line) and the
actual values (red line) are relatively close in some places, but in many places,
especially in the peaks, the error is large, which in accordance with the analysis of
Figure 2. The forecast results within 5 months of the simple dynamical reconstruction
model in section3 are good, but the long term prediction results after 5 months
become bad and the error increases quickly. So this is why we have to introduce the
self-memorization principle to improve the long term prediction results.

We sincerely hope for your satisfaction with our revision. Thank you again for
your kind suggestion.

Again it is not clear how the correlation and MAPE statistics were calculated -
only one value is given, so presumably it is taken over all (720 months) forecast?

Responses: Good suggestions. In pervious paper we haven’t explained clearly
how the correlation and MAPE statistics in Fig.3 were calculated. It isn’t taken over
all (720 months) forecast when only one value is given (The forecast for such a long
time is not possible). The figure 3 merges the 60 experiments (each experiment is the
prediction of the 12 month similar as Fig.2) on one picture. The Fig.3 is equivalent to
60 experiments instead of the results of only one experiment, because the results of one experiment are not entirely representative. And through multiple cross experiments can more objectively reflect the forecast capability of our model. So the forecast results of 60 cross experiment (each experiment is the prediction of the 12 month as Fig.2) according to the time sequence can merger into a new time series (from Jan.1951-Dec.2010), and then the pearson correlation coefficient (CC) and the mean absolute percentage error (MAPE) can be calculated by the new prediction time series and the time series of the actual value based on the formula in the above answer of 4-2 problems. Actually, the CC and MAPE are the average of the prediction values of the 60 cross experiments. That's how the correlation and MAPE statistics were calculated in Fig. 3.

Now we have added the above explanation from line 294 to 300 in page14 for readers’ better understanding.

We sincerely hope for your satisfaction with our revision. Thank you again for your kind suggestion.

4-6. However the discussion in lines 310-312 suggest that individual 12 month forecasts were also evaluated. Overall the discussion of the forecast process and its validation in not clear.

Responses: Good suggestions. The CC and MAPE in Fig.3 are the average of the prediction values of the 60 cross experiments. But each MAPE value of the above 60 experiments is not the same and the difference between the maximum and the minimum MAPE value is quite large, which means that the prediction results of the
simple dynamical reconstruction model in section3 is not stable. So that is another reason why we need to introduce self-memorization principle to improve our model.

We sincerely hope for your satisfaction with our revision. Thank you again for your kind suggestion.

Some minor points

1. In line 170, all 4 data sets range from Jan 1951 to Jan 2010, yet in at least 4 places, Responses: Good suggestions. Now we have deleted the other 3 places about the description of the length of the data. And in previous paper, “all 4 data sets from Jan. 1951 to Jan. 2010” is mistake in writing. Now we revised as “The time series of all data were from Jan. 1951 to Dec. 2010, 720 months in total” from line129 to line130 in page6.

We sincerely hope for your satisfaction with our revision. Thank you again for your kind suggestion.

2. lines 292, 373, 402 and 416 forecasts are evaluated up to December 2010?

Responses: Good suggestions. In previous paper, “all 4 data sets from Jan. 1951 to Jan. 2010” is mistake in writing. Now we revised as “The time series of all data were from Jan. 1951 to Dec. 2010, 720 months in total.” So the lines 292, 373, 402 and 416 forecasts are surely evaluated up to December 2010.

We sincerely hope for your satisfaction with our revision. Thank you again for your kind suggestion.
3. lines 249-253. Why does normalising the raw values avoid the overfitting problem?

Responses: Good suggestions. Now we have revised the sentences” To avoid the overfitting problem, we used $x_{nor} = \frac{x - x_{min}}{x_{max} - x_{min}}$ to normalize the raw value of each of the four predictors, then we used the normalized value to model and forecast.” as “In order to eliminate the dimensionless relationship between variables, data standardization is to transform data from different orders of magnitude to the same order of magnitude, thus making the data comparable. So we used $x_{nor} = \frac{x - x_{min}}{x_{max} - x_{min}}$ to normalize the raw value of each of the four predictors, then we used the normalized value to model and forecast.” from line243 to line248 in page12.

We sincerely hope for your satisfaction with our revision. Thank you again for your kind suggestion.

4. line 254. What criterion is used to determine what are "weak items" with "small dimension coefficient".

Responses: Good suggestions. In the previous paper, we have neglected to explain the criterion is used to determine what are "weak items" with "small dimension coefficient". In order to quantitatively compare the relative contribution of each item of our model to the evolution of the system, we calculated the relative variance contribution.
The formula is as follows: \[ R_i = \frac{1}{n} \sum_{j=1}^{n} \left[ \frac{T_i^2}{T_{ij}} \right], i = 1, 2, ..., 14. \] Where \( n \) is the length of the data, \( T_i = a_1x_1 + a_2x_2 + ... + a_{14}x_{14} \) is the item in the equation. According to our previous research (Hong et al., 2007), the variance contribution of the real item reflecting the performance of the model has a large proportion, while the variance contribution of the false term is almost zero, so we delete the weak items of \( R_i < 0.01 \).

Now we have added the above explanation about the criterion is used to determine what are "weak items" from line250-257 in page12.

We sincerely hope for your satisfaction with our revision. Thank you again for your kind suggestion.

References:


5. line 280 "forecast performance ... was better" than what??

Responses: Good suggestions. Now we have revised the sentence “From Fig. 2, forecast performance of \( T_1 \) and \( T_2 \) within 5 months was better.” as “From Fig. 2, forecast performance of \( T_1 \) and \( T_2 \) within 5 months was good.”

We sincerely hope for your satisfaction with our revision. Thank you again for your kind suggestion.
Section 6.2 - Table 5 The values reported here do not make sense. By construction, EOFs (the spatial patterns) are orthogonal, and the PCs (the time series) are uncorrelated. L’Heureux et al report that the correlation between PC1 and PC2 (using the same HADISST data set) is 0.4 when the time series are detrended. This is the same value quoted in Table 5. Has T2 been detrended here also?

Responses: Good suggestions. In Table 5, the values reported here do not make sense. Now we have deleted the Table5. In previous paper, we don’t have detrended T2. We have just smoothed the SSTA field before EOF. But due to a careless mistake, we use the data of a prediction experiment of 12 months to calculate the correlation coefficient in Table 5 and this is a mistake. We should use all data from Jan.1951 to Dec.2010, a total of 720 months to calculate the correlation coefficient, so the correlation coefficients in Table 5 are not correct in our previous paper. Now we have recalculated with the right data. And after the time series are detrended, we have recalculated that the correlation between PC1 and PC2 is 0.4024, which is the similar as L’Heureux et al.

We sincerely hope for your satisfaction with our revision. Thank you again for your kind suggestion.

7. EOF1 is the canonical ENSO pattern, and its time series is strongly correlated with the standard Nino indices (L’Heureaux et al give a value of 0.94 between their first EOF and the Nino3.4 index). In turn the Nino3.4 index is strongly correlated to the SOI, so that
is difficult to see the correlation between T1 and the SOI being as small as the 0.4 given in Table 5. (This correlation is where the term ENSO i.e. El Nino - Southern Oscillation arises)

Responses: Good suggestions. In the answer of the previous question, we mentioned that because of a careless mistake, correlation coefficient in the table 5 formula is not correct. Now we have recalculated with the right data. In the answer to question 2, the correlation coefficient of T1 and SOI in table 2 is 0.773, which is consistent with the fact that the Nino3.4 index is strongly correlated to the SOI.

We sincerely hope for your satisfaction with our revision. Thank you again for your kind suggestion.

8. Acronyms need to be defined the first time they are used, e.g. EOF on lines 128-130

Responses: Good suggestions. Now we have defined Acronyms in the first time they are used.

We sincerely hope for your satisfaction with our revision. Thank you again for your kind suggestion.

9. Figure caption (line 912) for figure 1 in List of figures is incorrect, and different to that given with the figure itself (line 959).

Responses: Good suggestions. Now we have revised the figure caption (line 1027) for figure 1 in List of figures.

We sincerely hope for your satisfaction with our revision. Thank you again for
your kind suggestion.

10. References are incomplete; there are at least 15 references that are not cited in the text, and a number that are cited but referenced.

Responses: Good suggestions. Now we have revised the list of references carefully and make all the references complete.

We sincerely hope for your satisfaction with our revision. Thank you again for your kind suggestion.
Forecasting experiments of a dynamical-statistical model of the sea surface temperature anomaly field based on the improved self-memorization principle

Mei Hong\textsuperscript{1,2}, Ren Zhang\textsuperscript{1,2}, Xi Chen\textsuperscript{1}, Ren Zhang\textsuperscript{1,2}, Dong Wang\textsuperscript{3}, Shuanghe Shen\textsuperscript{2}, Xi Chen\textsuperscript{1}, and Vijay P. Singh\textsuperscript{4}

\textsuperscript{1}Collaborative Innovation Center on Forecast and Evaluation of Meteorological Disaster, Nanjing University of Information Science & Technology, Nanjing 210044, China

\textsuperscript{2}Institute of Meteorology and Oceanography, National University of Defense Technology, Nanjing 211101, China

\textsuperscript{3}Key Laboratory of Surficial Geochemistry, Ministry of Education; Department of Hydrosciences, School of Earth Sciences and Engineering, Collaborative Innovation Center of South China Sea Studies, State Key Laboratory of Pollution Control and Resource Reuse, Nanjing University, Nanjing 210093, China

\textsuperscript{4}Department of Biological and Agricultural Engineering, Zachry Department of Civil Engineering, Texas A & M University, College Station, TX 77843, USA

Corresponding authors address:

1. Ren Zhang, Research Centre of Ocean Environment Numerical Simulation, Institute of Meteorology and Oceanography, National University of Defense Technology, Nanjing 211101, China

E-mail: 254247175@qq.com

4-2. Xi Chen, Research Centre of Ocean Environment Numerical Simulation, Institute of Meteorology and Oceanography, National University of Defense Technology, Nanjing 211101, China
Abstract: With the objective of tackling the problem of inaccurate long-term El Niño Southern Oscillation (ENSO) forecasts, this paper develops a new dynamical-statistical forecast model of sea surface temperature anomaly (SSTA) field. To avoid single initial prediction values, a self-memorization principle is introduced to improve the dynamic reconstruction model, thus making the model more appropriate for describing such chaotic systems as ENSO events. The improved dynamical-statistical model of the SSTA field is used to predict SSTA in the equatorial eastern Pacific and during El Niño and La Niña events. The long-term step-by-step forecast results and cross-validated retroactive hindcast results of time series $T_1$ and $T_2$ are found to be satisfactory, with a Pearson correlation coefficient of approximately 0.80 and a mean absolute percentage error (MAPE) of less than 15%. The corresponding forecast SSTA field is accurate in that not only is the forecast shape similar to the actual field, but the contour lines are essentially the same. This model can also be used to forecast the ENSO index. The temporal correlation coefficient is 0.8062, and the MAPE value of 19.55% is small. The difference between forecast results in summer-spring and those in winter-autumn is not high.
indicating that the improved model can overcome the spring predictability barrier to some extent. Compared with six mature models published previously, the present model has an advantage in prediction precision and length, and is a novel exploration of the ENSO forecast method.

**Keywords:** Dynamical-statistical forecast model; self-memorization principle; sea surface temperature field; long-term forecast of ENSO

1. **Introduction**

The El Niño Southern Oscillation (ENSO), the well-known coupled atmosphere–ocean phenomenon, was firstly proposed by Bjerknes (1969). The ENSO phenomenon can influence regional and global climates, so the prediction of ENSO has received considerable public interest (Rasmusson and Carpenter, 1982; Glantz et al., 1991).

Over the past two to three decades, one might reasonably expect the ability to predict warm and cold episodes of ENSO at short and intermediate lead times to have gradually improved (Barnston et al., 2012). Many countries have been focusing on ENSO forecasts since the 1990s, and the ENSO forecast has become one of the important research topics in the International Climate Change and Predictability Research plan. The U.S. International Research Institute for Climate and Society, the U.S. Climate Prediction Centre, Japan Meteorological Agency, and European Centre for Medium-Range Weather Forecasting have developed different coupled atmosphere–ocean models to forecast ENSO (Saha et al., 2006; Molteni et al., 2007).
The forecast models can generally be divided into two types (Palmer et al., 2004). The first type is typified by a dynamic model, which mathematically expresses physical laws that govern how the ocean and the atmosphere interact. The second type is typified by a statistical model, which requires large amounts of historical data and analyses the data to do forecasting (Chen et al., 1995; Moore et al., 2006).

Over the past three decades, ENSO predictions have made remarkable progress, reaching a stage where reasonable statistical and numerical forecasts (Jin et al., 2008) can be made 6–12 months in advance (Wang et al., 2009). However, there are three problems remaining to be resolved (Zhang et al., 2003a): (1) The current ENSO predictions are mainly limited to the short term, such as annual and seasonal predictions; (2) Although the representation of ENSO in coupled models has advanced considerably during the last decade, several aspects of the simulated climatology and ENSO are not well reproduced by the current generation of coupled models. The systematic errors in SST are often very large in the equatorial Pacific, and model representations of ENSO variability are often weak and/or incorrectly located (Neelin et al. 1992; Mechoso et al. 1995; Delecluse et al. 1998; Davey et al. 2002). (3) Coupled models of ENSO predictions initialized from observed initial states tend to adjust towards their own climatological mean and variability, leading to forecast errors. The errors associated with such adjustments tend to be more pronounced during boreal spring, which is often called the ‘‘spring predictability barrier’’ (Webster et al., 1999). More efficient models are therefore desired (Belkin and Niyogi, 2003; Weinberger and Saul, 2006). Therefore, the idea of combining
dynamical and statistical methods to improve weather and climate prediction has been
developed in many studies (Chou, 1974; Huang et al., 1993; Yu et al., 2014a; Yu et
al., 2014b). By introducing genetic algorithms (GAs), Zhang et al. (2006) inverted and
reconstructed a new dynamical-statistical forecast model of the tropical Pacific sea
surface temperature (SST) field using historic statistical data (Zhang et al., 2008).
However, there is one flaw in the forecast model: the time-delayed SST field. This is
because ENSO is a complicated system with many influencing factors. To overcome
information insufficiency in the forecast model, Hong et al. (2014) selected the
tropical Pacific SST, SSW and SLP fields as three modelling factors and utilized the
GA to optimize model parameters.

However, the above dynamical prediction equations which were proposed by
Hong et al. (2014), greatly depend on a single initial value, creating long-term
forecasts over 8 months that diverged significantly. These unsatisfactory results
indicate that this model needs to be improved. Cao (1993) first proposed the
self-memorization principle, which transforms the dynamical equations with the
self-memorization equations, wherein the observation data can determine the memory
coefficients. This method has been widely used in forecast problems in environmental,
hydrological and meteorological fields (Feng et al., 2001; Gu, 1998; Chen et al.,
2009). The method can avoid the question of initial conditions for the differential
equations, so it can be introduced here to improve the proposed dynamical forecast
model.

Therefore, an improved dynamical-statistical forecast model of the SST field
and its impact factors with a self-memorization function was developed. The improved model can absorb the information from past observations.

This paper is organized as follows: Research data and forecast factors are introduced in section 2. In Section 3 the reconstruction of the dynamical model of SSTA field is described. To improve the reconstruction model, the self-memorization principle is introduced in Section 4. Model forecast experiments are described in Section 5, and conclusions are given in Section 6.

2. Research data and forecast factors

2.1 Data

The monthly average SST data— from January 1951 to January 2010, 720 months in total— were obtained from the UK Met Office Hadley Centre for the region (30°S-30°N; 120°E -90°W). The gridded 1° × 1° Met Office Hadley Sea Ice and SST dataset (HadISST1; Rayner et al. 2003) includes both in situ and available satellite data. The sea areas provide important information on ocean-atmosphere coupling in the East and West Pacific Ocean and the El Niño / La Niña events. The reanalysis data, zonal winds and sea level pressures were obtained from the National Center for Environmental Forecast of America and the National Center for Atmospheric Research (Kalnay et al., 1996). The sea surface height (SSH) field was obtained from Simple Ocean Data Assimilation (SODA) data (James and Benjamin, 2008). Outgoing longwave radiation (OLR) was obtained from the National Oceanic and Atmospheric Administration (NOAA) satellites, at a resolution of 0.5° × 0.5° (Liebmann and Smith, 1996). The sea areas provide important information on
ocean-atmosphere coupling in the East and West Pacific Ocean and the El Niño and La Niña events. The reanalysis data and zonal winds were obtained from the National Center for Environmental Forecast (NCEP) of America and the National Center for Atmospheric Research (NCAR) (Kalnay et al., 1996). The Southern Oscillation Index (SOI) data were obtained from the Climate Prediction Center (CPC). The time series of all data were from Jan. 1951 to JanDec. 2010, 720 months in total.

2.2 EOF deconstruction

The sea surface temperature anomaly (SSTA) field can be calculated from the SST field and can be deconstructed into time (coefficients)-space (structure) using the empirical orthogonal function (EOF) method. Detailed information on the EOF method can be seen in the related references (Dommenget & Latif, 2002). We have used covariance matrix, because the covariance matrix was selected to diagnose the primary patterns of co-variability in the basin-wide SSTs, rather than the patterns of normalized covariance (or correlation matrix).

We used the smooths function with MATLAB to smooth the SSTA field before the EOF deconstruction, which is five points two times moving, mainly filtering out some noise points and outliers. Then an empirical orthogonal function (EOF) analysis of smoothed anomalies was performed, and the first two SSTA EOFs are shown in Figs. 1a and 1c. The principal component (PC) time series corresponding to the first and second EOFs are shown in Figs. 1b and 1d. The first EOF pattern, which accounted for 61.33% of the total SSTA variance, represented the mature ENSO phase...
(El Niño or La Niña), and the corresponding PC time series was highly correlated
(with a correlation coefficient of 0.85) with the cold tongue index (SST anomaly
averaged over 4°S–4°N, 180°–90°W) over the whole period. The second EOF,
accounting for 14.52% of the total SSTA variance, indicated the ENSO signal
beginning to enhance. The ENSO signal beginning to decay. Compared with the first
mode, these were slightly attenuated in terms of the scope and intensity. The above
analysis is similar to the EOF analysis of the SSTA field in the previous studies
(Johnson et al., 2000; Timmermann et al., 2001). This indicates that the front two
variance contribution modes can describe the main characteristics of the SSTA field
and El Niño/La Niña. Therefore, we can choose the $T_1, T_2$ time series EOF
decomposition modes as the modelling objects.

2.3 Selection of other prediction model factors

Considering the complexity of computation, the amount of variables in the
equations of our model can’t be too large, usually 3 or 4 for the best. This has been
explained in our previous studies (Zhang et al., 2006; Zhang et al., 2008). If there are
more than 4 variables in the modeling equation, it will cause the amount of
parameters such as $a_1, a_2, a_3, b_1, b_2, b_3, \ldots$ too large. The huge computation makes it
difficult to be precisely modeled. Thus, the total number of parameters in the model of
five variables was 102, which may cause an overfitting problem. Hence, when we
selected the model of five or six variables which entailed large amounts of
computation that made precision difficult, and too many parameters might cause an
overfitting phenomenon. If we choose only two or even fewer variables, the forecast
performance is poor too. Too few variables cause too small reconstructed parameters, resulting in amounts of important information missing out in the model. Thus, four variables are best for dynamically and accurately modeling. Because we have chosen two time series in section 2.2 as the modeling objects, now we should select the other two ENSO intensity impact factors.

The ENSO intensity impact factor is an important issue in ENSO prediction. Previous studies have been completed in this area, which found that teleconnection patterns, temperature, precipitation, wind and SSH may affect ENSO strength. For example, Trenberth et al. (1998) noted that PNA, SOI and OLR in the Pacific Intertropical Convergence Zone (ITCZ) are all closely related to ENSO. Webster(1999) pointed out after the 1970, Indian Ocean dipole (IOD) is not only affected by ENSO, but also affected the strength of ENSO (Ashok et al., 2001). Yoon and Yeh (2010) reported that the Pacific Decadal Oscillation (PDO) disrupts the linkage between El Niño and the following Northeast Asian summer monsoon (NEASM) through inducing the Eurasian pattern in the mid-high latitudes. The vast majority of studies (Tomita and Yasunari, 1996; Zhou and Wu, 2010; Kim et al., 2017) have concentrated on the impacts of ENSO on the East Asian winter monsoon (EAWM). During the EAWM season, ENSO generally reaches its mature phase and has the most prominent impact on the climate. Wang et al. (1999a) and Wang et al. (1999b) suggested that the zonal wind factors in the eastern and western equatorial Pacific play a critical role in the phase of transition of the ENSO cycle, which could excite eastward propagating Kelvin waves and affect the SSTA in the...
equatorial Pacific. Zhao et al. (2012) analyzed the characteristics of the tropical Pacific SSH field and its impact on ENSO events.

Based on the above analysis, we have selected nine factors, which may be closely related with the ENSO index (Niño3.4).

(1) The zonal wind in the eastern equatorial Pacific factor (u1) was calculated as the grid-point average of zonal wind in the area [5° S ~ 5° N, 150° W ~ 90° W].

(2) The zonal wind in the western equatorial Pacific factor (u2) was calculated as the grid-point average of zonal wind in the area [0° ~ 10° N; 135° E ~ 180° E].

(3) The PNA teleconnection factor was obtained from the CPC.

(4) the dipole mode index factor (DMI) was obtained from SSTA for June-July-August (JJA) based on Saji(1999) method.

(5) The SOI factor was obtained from the CPC.

(6) The PDOI factor was obtained from department of Atmospheric Sciences in the university of Washington. The web is http://tao.atmos.washington.edu/pdo/RDO.latest.

(7) The EAWM index (EAWMI) factor was proposed by Yang et al. (2002), which is defined by the meridional 850-hPa winds averaged over the region (20° ~40°N, 100°~140°E).

(8) The OLR in the ITCZ factor was calculated as the grid-point average of OLR in the area [10°N~20°N, 120°E~150°E].

(9) The SSH factor was calculated as the grid-point average of the SSH data in the area [10° S ~ 10° N; 120° E ~ 60° W].
A correlation analysis of the above factors was carried out and the results are shown in Table 1. Table 1 shows that SOI and EAWMI have the stronger correlation with the front two time series $t_1, t_2$ than the other 7 factors. The results are also consistent with previous research (Clarke and Van Gorder, 2003; Drosdowsky, 2006; Zhang et al., 1996; Wang et al., 2008; Yang and Lu, 2014). Therefore, the first time series $t_1$, the second time series $t_2$, SOI and EAWMI will be selected as prediction model factors.

The ENSO intensity impact factor is an important issue in the ENSO prediction. Previous studies have found that teleconnection patterns, temperature, precipitation, wind and SSH may affect the ENSO strength (Trenberth et al., 1998; Webster, 1999; Ashok et al., 2001; Yoon and Yeh, 2010; Tomita and Yasunari, 1996).

For example, Trenberth et al. (1998) noted that the Pacific North American Oscillation Index (PNA) and SOI in the Pacific Intertropical Convergence Zone (ITCZ) were all closely related to ENSO. Liao et al. (2007) also noted that the decadal variation during ENSO events had a close relationship with the SOI index. The vast majority of studies (Tomita and Yasunari, 1996; Zhou and Wu, 2010) have concentrated on the impacts of ENSO on the East Asian winter monsoon (EAWM). During the EAWM season, ENSO generally reaches its mature phase and has the most prominent impact on the climate. Wang et al. (1999a) and Wang et al. (1999b) suggested that the zonal wind factors in the eastern and western equatorial Pacific played a critical role in the transition phase of the ENSO cycle, which could excite eastward propagating Kelvin waves and affect the SSTAs in the equatorial Pacific.
Based on the above analysis, we selected four factors, which may be closely related with the ENSO index (Niño 3.4) and were obtained as follows:

1. The zonal wind in the eastern equatorial Pacific factor ($u_1$) was calculated as the grid-point average of zonal wind in the area [$5^\circ$ S ~ $5^\circ$ N, $150^\circ$ W ~ $90^\circ$ W].

2. The PNA teleconnection factor was obtained from the CPC.

3. The SOI factor was obtained from the CPC.

4. The EAWM index (EAWMI) factor was proposed by Yang et al. (2002), which is defined by the meridional 850-hPa winds averaged over the region ($20^\circ$ N ~ $40^\circ$ N, $100^\circ$ ~ $140^\circ$ E).

All the four data selected ranged from January 1951 to January 2010.

Actually, how many variables and which variables are used in our model become a key issue to be resolved. We can introduce a stepwise regression principle to choose more reasonable predictors (Yim et al., 2015), because the stepwise procedure can help selecting statistically important predictors at each step. The significance of each predictor selected was based on its significance in increasing the regressed variance by the standard $F$ test (Panofsky and Brier, 1968). A 95% statistical significance level was used as a criterion to select a new predictor at each step. Once selected into the model, a predictor can only be removed if its significance level falls below 95% by the addition/removal of another variable. For example, for the model of only one variable, because we forecast the ENSO index, we should choose $u_1$ as the variable. Considering that $u_1$ accounts for 61.33% of the total SSTA variance, so we chose $-u_1$ as the variable. For the model of two variables, there...
are five factors ($\text{Tu}$, PNA, SOI and EAWMI) which can be chosen for the second variable. Taking advantage of the stepwise regression ideas and selecting statistically important predictors by a standard $F$ test, we can find the largest $F$ test value among the five factors. That is $T_2$. Continuing this step, we can also select the reasonable factors for the model of three variables. Based on this thought, when the number of variables is determined, we can choose the most statistically important variables to reconstruct the prediction model. The forecast results of these models can be seen in table 1.

From table 1, the forecast results of all six models are satisfactory, where the temporal correlations of the models are all greater than 0.60 and the root mean square errors are all less than 0.81. Among all six models, the forecast results of four variables are the best for the following reasons:

(1) In general, the amount of parameters is less than 10% of the sample size, which can avoid over-fitting (Tetko et al., 1995). The number of parameters of the model of four variables $T_1 T_2 \text{SOI} \text{EAWMI}$ is 56, but we deleted the parameters which contributed little to the prediction. That means that there are 56 parameters in equation (1) in section 3, but there are only 34 parameters in equation (3) in section 3 which is our final prediction equation. In section 5.1, because $\phi$ is identified as 6, the number of parameters of the self-memory function $\phi$ is 28. Therefore, the total number of parameters in the model of four variables is 62, which is less than 10% of the sample size (720 months).
of five variables is 100. Although the parameters which contributed a little were deleted, the number was still 72, and the number of self-memorization parameters was 30 ($p_d$ determined as 5). Thus, the total number of parameters in the model of five variables was 102, which was more than 10% of the sample size (720 months). This will cause an overfitting problem. Hence, when we selected the model of five or six variables which entailed large amounts of computation that made precision difficult, and too many parameters caused an overfitting phenomenon. That is why the forecast results of five or six variables were worse than those of four variables.

(2) The models of one, two, and three variables can avoid the overfitting problem, but too few variables will result in too few reconstruction parameters, causing important information missing from the model. Especially, when the model of one or two variables was considered, we only studied the self-memorization of the ENSO system but did not consider the mutual memorization between factors. Thus, equations of our model only contained a self-memory term, not an exogenous effect term. That is why the forecast results of one, two, and three variables were worse than those of four variables.

Based on the above analysis, we finally chose $x_{t-1}, x_{t-2}, \text{SOI}$, and $\text{EAWMI}$ as predictors for the model.

### 3. Reconstruction of dynamical model based on GA

Takens’ delay embedding theorem (Takens, 1981) provides the conditions under which a smooth attractor can be constructed from observations made with a generic
function. Later results replaced the smooth attractor with a set of arbitrary box-counting dimensions and the class of generic functions with other classes of functions. Takens had shown that if we measured any single variable with sufficient accuracy for a long period of time, it would be possible to construct the underlying dynamical structure of the entire system from the behavior of that single variable using delay coordinates and the embedding procedure. It was therefore possible to construct a dynamical model of system evolution from the observed time series. Introducing this idea here, four time series of the $T_1$, $T_2$, SOI and EAWMI factors were chosen to construct the dynamical model.

The basic idea of statistical-dynamical model construction is discussed in Appendix A and was introduced in our previous work (Zhang et al., 2006; Hong et al., 2014).

A simplified second-order nonlinear dynamical model can be used to depict the basic characteristics of atmosphere and ocean interactions (Fraedrich, 1987). Suppose that the following nonlinear second-order ordinary differential equations are taken as the dynamical model of reconstruction. In the equations, $x_1, x_2, x_3, x_4$ were used to represent the time coefficient series of $T_1$, $T_2$, SOI and EAWMI.

\[
\frac{dx}{dt} = a_1 x_1 + a_2 x_2 + a_3 x_3 + a_4 x_4 + a_5 x_1^2 + a_6 x_2^2 + a_7 x_3^2 + a_8 x_4^2 + a_9 x_1 x_2 + a_{10} x_1 x_3 + a_{11} x_1 x_4 + a_{12} x_2 x_3 + a_{13} x_2 x_4 + a_{14} x_3 x_4
\]

\[
\frac{dx}{dt} = b_1 x_1 + b_2 x_2 + b_3 x_3 + b_4 x_4 + b_5 x_1^2 + b_6 x_2^2 + b_7 x_3^2 + b_8 x_4^2 + b_9 x_1 x_2 + b_{10} x_1 x_3 + b_{11} x_2 x_3 + b_{12} x_2 x_4 + b_{13} x_3 x_4 + b_{14} x_3 x_4
\]

\[
\frac{dx}{dt} = c_1 x_1 + c_2 x_2 + c_3 x_3 + c_4 x_4 + c_5 x_1^2 + c_6 x_2^2 + c_7 x_3^2 + c_8 x_4^2 + c_9 x_1 x_2 + c_{10} x_1 x_3 + c_{11} x_2 x_3 + c_{12} x_2 x_4 + c_{13} x_3 x_4 + c_{14} x_3 x_4
\]

\[
\frac{dx}{dt} = d_1 x_1 + d_2 x_2 + d_3 x_3 + d_4 x_4 + d_5 x_1^2 + d_6 x_2^2 + d_7 x_3^2 + d_8 x_4^2 + d_9 x_1 x_2 + d_{10} x_1 x_3 + d_{11} x_2 x_3 + d_{12} x_2 x_4 + d_{13} x_3 x_4 + d_{14} x_3 x_4
\]
Based on the parameter optimization search method of GA in Appendix A, the
time coefficient series of $T_1$, $T_2$, SOI and EAWMI from January 1951 to April 2008
are chosen as the expected data to optimize and retrieve model parameters. In order to
eliminate the dimensionless relationship between variables, data standardization is to
transform data from different orders of magnitude to the same order of magnitude.
thus making the data comparable. So we used $x_{nor} = \frac{x - x_{\min}}{x_{\max} - x_{\min}}$ to normalize the raw
value of each of the four predictors, then we used the normalized value to model and forecast. To avoid the overfitting problem, we used $x_{nor} = \frac{x - x_{\min}}{x_{\max} - x_{\min}}$ to normalize the raw value of each of the four predictors, then we used the normalized value to model and forecast. Finally, we made forecast results revert back to the raw data magnitude by $x = x_{nor}(x_{\max} - x_{\min}) + x_{\min}$.

In order to quantitatively compare the relative contribution of each item of our model to the evolution of the system, we calculated the relative variance contribution.
The formula is as follows: $R_i = \frac{1}{n} \sum_{j=1}^{n} \frac{T_i^2}{\sum_{i=1}^{n} T_i^2}$, Where $n$ is the length of the data, $T_i = a_1 x_1, a_2 x_2, \ldots, a_1 x_1 x_4$ is the item in the equation. According to our previous research (Hong et al., 2007), the variance contribution of the real item reflecting the performance of the model has a large proportion, while the variance contribution of the false term is almost zero, so we delete the weak items of $R_i < 0.01$. 

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After deleting the weak items with small dimension coefficients, the nonlinear dynamical model of the first time series $r_1$, the second time series $r_2$, SOI and EAWMI can be reconstructed as follows:

$$\frac{dx_1}{dt} = F_1 = -0.3328x_1 + 1.2574x_2 - 0.3511x_3 - 0.0289x_4^2 + 3.1280x_4^3 + 0.0125x_4^4 + 2.7805x_5x_6 - 1.5408x_5x_6$$

$$\frac{dx_2}{dt} = F_2 = 1.0307x_1 - 3.1428x_2 + 0.3095x_3 + 4.2301x_4^2 - 1.2066x_4^3 + 2.5024x_4^4 - 0.2891x_5x_6 + 0.7815x_5x_6 - 0.4266x_5x_6$$

$$\frac{dx_3}{dt} = F_3 = -2.3155x_1 + 3.2166x_2 + 1.5284x_3 - 1.4527x_4^2 - 0.0034x_4^3 - 4.1206x_4^4 - 0.0025x_5x_6 + 0.0277x_5x_6 + 1.2860x_5x_6$$

$$\frac{dx_4}{dt} = F_4 = 0.4478x_1 - 0.0268x_2 + 0.8995x_3^2 - 2.3890x_3^3 + 2.037x_3^4 + 1.3035x_4x_5 + 2.0458x_4x_5 - 2.0015x_4x_5$$

The appropriate model coefficient estimates determine the robustness of the model and the accuracy of forecast results. We should now judge whether the model coefficients are appropriate or not.

First, the largest Lyapunov exponent (LLE) is one of the indexes that can represent the characteristics of chaotic systems. The final Lyapunov exponents of Eq. (2) were $[0.0433, -0.0012, -0.1285]$, containing both a negative Lyapunov exponent and two positive Lyapunov exponents, which demonstrate that our dynamic system is indeed a chaotic system.

Second, we calculated the equilibrium roots of Eq. (2). Only the third equilibrium was adjudged to be stable, based upon higher-order terms within the Taylor series, the indices of which were mostly in accordance with the actual weather system. The indices in the unstable equilibria could not accurately describe the actual weather. Based on these two aspects, we can see that the model coefficient estimates were reasonable and reflected the dynamical characteristics of the model.

The model required testing. Because the training period was from January 1951
to April 2008, we chose $T_1$, $T_2$, SOI and EAWMI of May 2008, which were not used as initial forecast data in the modeling. Next, the Runge–Kutta method was used to do the numerical integration of the above equations, and every step of the integration was regarded as 1 month’s worth of forecasting results. As a result, forecast results of four time series over a period of 20 months were obtained. Here, the focus was on the forecast results of $T_1$ and $T_2$, as shown in Fig. 2.

The Pearson correlation coefficient (CC) (Wang et al. 2009b) and the mean absolute percentage error (MAPE) (Hu et al. 2001) are employed as objective functions to calibrate the model. The CC evaluates the linear relationship between the observed and predicting values and MAPE measures the difference between the observed and predicting values.

From Fig. 2, forecast performance of $T_1$ and $T_2$ within 5 months was better. Using $T_1$ as an example, the temporal correlation between model predictions and corresponding observations over the first five months forecasts was 0.8966 and the mean absolute percentage error (MAPE) (Hu et al., 2001),

$$\text{MAPE} = \frac{1}{n} \sum_{i=1}^{n} \left| \frac{D_i(i) - D_o(i)}{D_o(i)} \right| \times 100, (n = 5),$$

was 8.32%. However, after 5 months, MAPE increased rapidly, and was 31.29% at 10 months. The model forecast then significantly diverged from observations, and the forecast became inaccurate. After 10 months, the forecast results became increasingly worse, which indicated that the forecast of the model after 5 months was unacceptable. The forecast results of $T_2$ were similar to those of $T_1$.

The model’s skill should be further assessed by cross-validated retroactive
hindcasts of the time series. As in the above example, omitting a portion of the time series (12 months, January-Jan, 1951 to January-Dec, 1952) from observations, we trained the model based on the data from February-Jan, 1952-1951 to December-Dec, 2010, and then predicted the omitted segments (12 months, Jan, 1951 to Dec, 1951 to January 1952). Then in the next prediction experiment, the omitted segment is Jan, 1952 to Dec, 1952 and the training samples are Jan, 1951 to Dec, 1951 and Jan, 1953 to Dec, 2010. So the forecast time series is Jan, 1952 to Dec, 1952. We then repeated this procedure by moving the omitted segment along the entirety of the available time series. Each experiment has used the different training sample and have established the different model equation (but the method is the same). The similar process of the cross-validated retroactive hindcasts has also been used in the previous literatures (Hu et al., 2017).

Finally, we obtained cross-validated retroactive hindcast results of $T_1$ and $T_2$, as shown in Fig. 3. So the forecast results of 60 cross experiment (each experiment is the prediction of the 12 month as Fig.2) according to the time sequence can merger into a new time series (from Jan, 1951-Dec, 2010), and then the pearson correlation coefficient (CC) and the mean absolute percentage error (MAPE) can be calculated by the new prediction time series and the time series of the actual value. Figure 3 is combined results of the 60 forecast experiments.

As Fig. 2, the forecast performance of $T_1$ and $T_2$ in Fig. 3 was not satisfactory. The model forecast significantly diverged from observations, and the forecast became inaccurate. The temporal correlations CC of $T_1$ and $T_2$ between model predictions and corresponding observations were 0.3411 and 0.4176, respectively. Additionally, the
mean absolute percentage errors (MAPE) of $T_1$ and $T_2$ were 65.42% and 57.56%, respectively. This indicates that the forecast of the model in the long-term was inaccurate and unacceptable.

The forecast result may be inaccurate when the integral forecasting time is long. There will be a significant divergence which will cause an ineffective forecast. To improve the forecast accuracy, the forecast not only depends on the integral equation but also on a single initial value. Choosing the different initial value will cause different forecast accuracy. For example, in a total of 60 cross-validated retroactive hindcasts examples, the minimum MAPE was 37.65%, while the maximum MAPE was 89.88%. A forecast, depending on a single initial value, will cause instability of the forecast results. These two problems are addressed by introducing the self-memorization principle in the next section.

4. Introduction of self-memorization dynamics to improve the reconstructed model

In the above discussion, it was shown that the accuracy of the forecast results of equation (2) were unsatisfactory. To improve long-term forecasting results, the principle of self-memorization can be introduced into the mature model (Gu, 1998; Chen et al., 2009). The principle of self-memorization dynamics (Cao, 1993; Feng et al., 2001) can be seen in Appendix B.

Based on Eq. (B10) in Appendix B, the improved model can be expressed as
where \( y_i \) is replaced by the mean of two values at adjoining times; i.e.,

\[
y_i = \frac{1}{2} (x_{i+1} + x_i)
\]

\( F \) is the dynamic core of the self-memorization equation, which can be obtained from Eq. (2); and \( \alpha \) and \( \theta \) are the memory coefficients, the formula for which can be found in Appendix B.

If the values of \( \alpha \) and \( \theta \) can be obtained, Eq. (3) can be used to obtain the results of final prediction. The memory coefficients \( \alpha \) and \( \theta \) in Eq. (3) were calibrated using the least-squares method with the same data (January 1951 to April 2008) as those used in Section 3. Eq. (3) can be deconstructed as follows (\( M \) is the length of the time series):

\[
\begin{pmatrix}
    x_{i_1} \\
    x_{i_2} \\
    \vdots \\
    x_{i_M}
\end{pmatrix} = \begin{pmatrix}
    \alpha_{p-1} \\
    \alpha_p \\
    \vdots \\
    \alpha_{-1}
\end{pmatrix} + \begin{pmatrix}
    y_{i-p-1} & y_{i-p,1} & \ldots & y_{i-p,1} \\
    y_{i-p,2} & y_{i-p,2} & \ldots & y_{i-p,2} \\
    \vdots & \vdots & \ddots & \vdots \\
    y_{i-1,M} & y_{i-1,M} & \ldots & y_{i-1,M}
\end{pmatrix} \begin{pmatrix}
    \theta_{p-1} \\
    \theta_{p} \\
    \vdots \\
    \theta_{0}
\end{pmatrix}
\]

\[
F = \begin{pmatrix}
    F_{-p,1} & F_{-p+1,1} & \ldots & F_{0,1} \\
    F_{-p,2} & F_{-p+1,2} & \ldots & F_{0,2} \\
    \vdots & \vdots & \ddots & \vdots \\
    F_{-p,M} & F_{-p+1,M} & \ldots & F_{0,M}
\end{pmatrix}
\]
The matrix equation is:

\[ X = Y\alpha + F\theta \]  \hspace{1cm} (4)

where \( Z = [Y; F] \), \( W = \begin{bmatrix} \alpha \\ \vdots \\ \Theta \end{bmatrix} \).

Eq. (4) can be written as:

\[ X = ZW \]  \hspace{1cm} (5)

The memory coefficients vector \( W \) can be calibrated using the least squares method:

\[ W = (Z^T Z)^{-1} Z^T X \]  \hspace{1cm} (6)

The memory coefficients \( a, \theta \) can be obtained from Eq. (6). We then made a prediction using the self-memorization equation (3), which used the \( p \) values before \( t_0 \).

The coefficients in F and W were used with the same training data from January 1951 to April 2008. In the forecast examples, we trained both the coefficients in F and W at the same time, but in the paper we describe them separately to facilitate the reader for better understanding.

5. Model prediction experiments

5.1 Forecast of time series \( T_1 \) and \( T_2 \)

The training sample for the model was from January 1951 to April 2008. Here, from Eq. (3), the forecast results using \( r, r_1, \) SOI and EAWMI factors can be calculated, called as step-by-step forecast.

When the retrospective order \( p \) is confirmed, step-by-step forecasts can be
carried out. For example, when the \( r, r_i \), SOI and EAWMI values of May 2008 were forecast, \( y_i \) was obtained from the previous \( p + 1 \) time of \( r, r_i \), the SOI and the EAWMI data, and \( F(x_i, x_{i+1}, x_{i+2}, x_{i+3}) \) was obtained from the previous \( p \) times of \( r, r_i \), the SOI and the EAWMI data. All four equations were integrated simultaneously. Taking these in Eq. (3), we can get the \( r, r_i \), SOI and EAWMI values of May 2008, which can be taken as the initial values for the next prediction step. Then, the \( r, r_i \), SOI and EAWMI values from June 2008 and so on, can be generated.

5.1.1 Determination of \( p \)

Based on the self-memorization principle, the self-memorization of the system determines the retrospective order \( p \) (Cao, 1993). If the system forgets slowly, parameters \( a \) and \( \theta \) will be small and the \( p \) value should be high. The SSTA field forecasts were on a monthly scale, the change of which was slow in contrast to large-scale atmospheric motion. So parameters \( a \) and \( \theta \) were small, and generally, the \( p \) value was in the range 5 to 15.

The retrospective order \( p \) was obtained by a trial calculation method. We selected the \( p \) values in the range 4 to 16 to construct the model. The correlation coefficients \( \text{CC} \) and MAPE of long-term fitting test (from February 1951 to December 2010) are shown in Table 2, which can be used as the standard to determine the retrospective order \( p \).

Table 2 indicates that when \( p = 6 \), the MAPE values of long-term fitting test were the smallest and the correlation coefficients \( \text{CCs} \) were the largest. Also, when \( p \) from 5 to 9, \( \text{CCs} \) were all more than 0.58 and the forecast results were all...
good, which is consistent with our interpretation of the physical mechanisms in section 6.2 below. SOI and EMWMI were 5-12 months lead relationships with SST (Xu et al., 1993; Chen et al., 2010; Wang et al., 2003). Using a cumulative period of SOI-, EMWMI 5-8 months ahead as initial values can help improve the final forecast results. Our results in table 2 are consistent with the actual physical ENSO process. Therefore, we selected the retrospective order as $p=6$.

Then, the prediction experiments can be carried out, based on improved self-memorization Eq. (3).

The improved self-memorization equation of $t_i, t_{i-1}, SOI$ and EAWMI can then be established. After the differential equation was discretely dealt with, the memory coefficients were solved by the least-squares method given in section 4 (Training period is January 1951 to April 2008). Finally, the improved prediction equation of $t_i, t_{i-1}, SOI$ and EAWMI, based on the self-memorization principle, can be expressed as:

\[
\begin{align*}
\alpha_{i} &= \sum_{i=1}^{1} \alpha_{i} y_{i} + \sum_{i=0}^{6} \theta_{i} f_{1}(x_{i}, x_{i-1}, x_{i-2}, x_{i-3}) \\
\alpha_{2i} &= \sum_{i=1}^{1} \alpha_{2i} y_{2i} + \sum_{i=0}^{6} \theta_{2i} f_{2}(x_{i}, x_{i-1}, x_{i-2}, x_{i-3}) \\
\alpha_{3i} &= \sum_{i=1}^{1} \alpha_{3i} y_{3i} + \sum_{i=0}^{6} \theta_{3i} f_{3}(x_{i}, x_{i-1}, x_{i-2}, x_{i-3}) \\
\alpha_{4i} &= \sum_{i=1}^{1} \alpha_{4i} y_{4i} + \sum_{i=0}^{6} \theta_{4i} f_{4}(x_{i}, x_{i-1}, x_{i-2}, x_{i-3})
\end{align*}
\]  

where
A step-by-step forecast was performed. The retrospective order $p = 6$ means that earlier seven observation data ($p + 1 = 7$) should be used during the forecasting process. The forecast results per month were saved for the next period predictions.

5.1.2 Long-term step-by-step forecasts of $T_1$ and $T_2$

To test the actual forecast performance of the above improved model, long-term step-by-step forecasts of $T_1$ and $T_2$ from May 2008 to December 2010 for 20 months were carried out, as shown in Fig. 4. The forecast results of $T_1$ and $T_2$ were good. Within 8 months, the correlation coefficients $CC$s of $T_1$ and $T_2$ were 0.9163 and 0.9187. MAPEs of $T_1$ and $T_2$ were small, only 5.86% and 6.78%. The forecast time series from 8 months to 14 months gradually diverged, but the trend was acceptable. The CC correlation coefficients of $T_1$ and $T_2$ reached 0.8375 and 0.8251, and MAPEs of $T_1$ and $T_2$ were 8.32% and 9.11%. After 14 months, forecast began to diverge and the error started to increase, but the correlation CC coefficients of $T_1$ and $T_2$ remained about 0.6899 and 0.6782, and MAPEs reached 18.31% and 19.44%, which can be acceptable.

5.2 Cross-validated retroactive hindcasts of time series $T_1$ and $T_2$
As in section 3, the model’s skill should be further assessed by cross-validated retroactive hindcasts of the time series. Because our step-by-step forecasts need the earlier seven observation data \( (\rho + 1 = 7) \), we can obtain cross-validated retroactive hindcast results of \( T_1 \) and \( T_2 \) from August 1951 to December 2010, as shown in Fig. 5.

From Fig. 5, the forecast performance of \( T_1 \) and \( T_2 \) was good. The correlation coefficients of \( T_1 \) and \( T_2 \) were 0.7124 and 0.7036, respectively. The MAPEs of \( T_1 \) and \( T_2 \) were small, only 19.57% and 19.79%, respectively. The peaks and valleys of \( T_1 \) and \( T_2 \) were also forecasted accurately. The forecast results indicated that the cross-validated retroactive hindcast results of \( T_1 \) and \( T_2 \) were close to the observed values. Compared to Fig. 3, the improved model had better forecast abilities than the original model.

Many researchers (Zhang et al., 2003b; Smith, 2004) have used Oceanic Niño Index (ONI) which is used by the U.S. NOAA Climate Prediction Center to determine the El Niño and La Niña years. It defined that the ONIs of five consecutive months in winter were all more than 0.5 (less than -0.5) is the ElNiño (La Niña) year. Based on the above criterion, we can divide the total 60 years (1951-2010) into three categories. It includes the 18 examples of ElNiño year (such as 1958, 1964, 1966, etc.), 22 examples of LaNiña year (such as 1951, 1955, 1956, etc.) and the remaining 20 experiments of the neutral year. Since the details in Fig.5 is not clear, we list the forecast results of 60 experiments (including 18 El Niño examples, 22 La Niña examples and 20 Neutral examples) in table 3.
From table 3, the average of \( \text{CC} \) of both \( T_1 \) and \( T_2 \) of 60 experiments within 6 months was more than 0.84 and MAPE was less than 8%. The average of \( \text{CC} \) within 12 months was more than 0.74 and MAPE was less than 12%. According to the literature (Barranel et al., 1999), when MAPE was less than 15%, which means the error was not great and the forecast results were good. Obviously, the forecast results of El Niño / La Niña experiments were a little worse than those of neutral experiments, which means the forecast ability of our model for the abnormal situation was a little worse than those for the normal situation. But even for El Niño / La Niña experiments, the average of \( \text{CC} \) was still more than 0.7 and MAPE was less than 15%, which means the error was not too large and was still within an acceptable range.

5.3 Forecast of the SSTA field

When we obtained the forecast results of the time coefficient series \( T_1 \) and \( T_2 \), we submitted them into the following equation to reconstruct the forecast SSTA field:

\[
\hat{x}_t = \sum_{n=1}^{12} E_n \cdot T_{nt} \quad t = 1, 2, ..., 12
\]

where \( E_n \), \( T_{nt} \) are the EOF space fields and forecast time coefficients, respectively, and \( \hat{x}_t \) is the forecast SSTA field reconstructed by EOF.

After reconstruction of the space mode (treated as constant) and time coefficient series (model prediction), the forecast of the SSTA fields was obtained, based on the forecast results of \( T_1 \) and \( T_2 \) in Section 5.2. For economy of space, we cannot draw all of the forecasted SSTA fields, so we selected a strong El Niño event (December 1997), a strong La Niña event (December 1999) and a neutral event (November 2002) as examples.
Fig. 6 shows the forecast SSTA field during a strong El Niño event. From the actual SSTA field in December 1997 (Fig. 6a), an obvious warm tongue structure occurred in the area of \([10^\circ S \sim 5^\circ N, 90^\circ W \sim 150^\circ W]\) in the Eastern Equatorial Pacific, and a warm anomalous distribution arose in the west Pacific, which indicated a weak El Niño event. The forecasted SSTA field of December 1997 is shown in Fig. 6b. Although the range of warm tongue was a litter bigger than the actual situation, the forecast shape was similar to the actual field and also the contour lines were similar. The average MAPE between the forecast field and the actual field is 8.56%, which was controlled within 10%. The forecast results of the improved model event were quite good for the El Niño event.

Fig. 7 shows the forecasted SSTA field of a strong La Niña event. From the actual SSTA field in December 1999 (Fig. 7a), an obvious cold pool occurred in the area of \([10^\circ S \sim 10^\circ N, 120^\circ W \sim 180^\circ W]\) in the Equatorial Pacific, which covered the Niño3.4 area. This SSTA field presented a strong strength La Niña event. The forecast SSTA field from December 1999 is shown as Fig. 7b. Although the strength of the cold pool was weaker than the actual situation, the forecast shape was similar to that of the actual field. The average MAPE between the forecast field and the actual field was 9.69%. The errors were larger than that of the El Niño event, but they can be controlled within 10%, which is acceptable.

Fig. 8 shows the forecasted SSTA field of a neutral event. From the actual SSTA field in November 2002 (Fig. 8a), a warm pool occurred in the area of \([10^\circ S \sim 10^\circ N, 120^\circ W \sim 180^\circ W]\) in the Equatorial Pacific, which covered the Niño3.4 area. However,
the warm pool was small and weak, which represented a neutral event. The forecasted SSTA field from November 2002 is shown in Fig. 8b. Comparing Figures 6, 7 and 8, we can see that the forecasted SSTA field of a neutral event was a little worse than that of the El Niño and La Niña events. The forecasted shape of the SSTA field basically described the actual situation, but the warm pool in the Niño3.4 area was stronger and bigger than that of the actual situation, which indicated a borderline El Niño event. The average MAPE between the forecasted field and the actual field was 14.50%, which was big but can be accepted.

We obtained the average values of MAPE of 18 El Niño events, 22 La Niña events and 20 neutral events, which were 9.52%, 9.88% and 14.67%, respectively, representing a good SSTA field forecasting ability of our model.

5.4 Forecast of ENSO index

The ENSO index can be represented as the sea surface temperature anomaly (SSTA) in the Niño-3.4 region (5° N-5° S, 120°-170° W) and the ENSO index forecast was the 3-month forecast (Barnston et al. 2012). So we also can pick up the ENSO index from the above forecasted SSTA field. The forecast results of the ENSO index within 20 months can also be obtained. The definition of lead time can be seen in the reference (Barnston et al. 2012). Therefore, similar to the forecast experiment in section 5.1, a succession of running 3-month mean SST anomalies with respect to the climatological means for the respective prediction periods, averaged over the Niño 3.4 region, can be obtained, as demonstrated in Fig. 9.

The evaluation criteria of the ENSO index is the temporal correlation (TC), its
The definition and specific calculation steps can be seen in these literatures (Kathrin et al., 2016; Nicosia et al., 2013). The TC is often used to measure the prediction effect of the ENSO index. For example, Barnston et al. in 2012 also used the TC to compare the forecast skill of 21 real-time seasonal ENSO models.

The forecast results within lead times of 18 months are shown in Fig. 9, which demonstrate that the forecast results of the ENSO index are good. Within lead time of 12 months, the correlation coefficient TC was 0.8985 and the MAPE value was small, only 8.91%. In addition, the borderline La Niña event in 2008–2009 was predicted well. After lead times of 12 months, forecasts began to diverge and the errors started to increase. Although the correlation coefficient TC remained approximately 0.61, MAPE reached 18.58%. Therefore, a moderate strength El Niño event that occurred in 2009/10 was not predicted.

We should give more examples to test the ENSO prediction ability of our model. As in section 5.3, we can divide 60 examples as three types, which are examples of El Niño year, La Niña year and neutral year. Finally, we can obtain the forecast results of different types of examples in different lead times, as shown in table 4.

From table 4, the average CC-TC of 60 experiments was 0.712 and the average MAPE was 7.62% within 12 months for all seasons of lead time, which indicates that the overall ENSO forecast ability of our model was good. The forecast results of the El Niño examples were significantly worse than those of La Niña examples, while the forecast results of La Niña examples were significantly worse than those of neutral examples, which show the model forecast ability of the abnormal state was worse than
the normal state of the ENSO index. Even for the forecast results of El Niño examples, the average CC was still above 0.6 and the average MAPE can be controlled below 10%, which means the forecast results were still in the acceptable range. Our model not only accurately predicted the stronger El Niño and La Niña phases but also the neutral states. But the forecast results in summer were a little worse than those in winter, as shown in Fig.10.

The ENSO forecast often had a spring predictability barrier (Webster, 1999), which was most prominent during decades of relatively poor predictability (Balmaseda et al., 1995). To test our model, the skill should be computed over the entire time series and separately for seasonal subsets of the time series. From the table4, we can see that the average cumulative correlation coefficient and MAPE of winter were compared with those of summer, as shown in Fig.10. The average cumulative correlation and average cumulative MAPE values between the forecast values and the actual values changed with time, from which good trends of forecast results can be seen. As long as the forecast time increased, the cumulative MAPE increased and the correlation decayed gradually. The forecast results appeared to diverge. Although the forecast results of the present model in the summer-spring were worse than in the winter-autumn, the margin was not high, which means the model can overcome the “spring predictability barrier,” to some extent.

5.5 Compared with six mature models

Barnston et al. (2012) compared many ENSO forecast models. Based on his research, we selected four high quality dynamical models, including ECMWF, JMA,
the National Aeronautics and Space Administration Global Modelling and
Assimilation Office (NASAGMAO) and the National Centre for Environmental
Prediction Climate Forecast System (NCEP CFS; Version1). Two high quality
statistical models also be selected, including the University of California, Los Angeles
Theoretical Climate Dynamics (UCLA-TCD) multilevel regression model and the
NOAA/NCEP/CPC constructed Analogue (CA) model. The detail of the above
models can be seen in these references (Reynolds et al., 2002; Luo et al.,
2005; Barnston et al., 2012).

We then compared the forecast ability of the above six models with that of our
model. All of the experiments of our model and six other models were conducted
under the same conditions using the same historical data for modelling and the same
initial values to forecast. In the CPC website, there are detailed explanations of six
models’ training samples and the initial values. So we do not need to install all these
models on their own machines and run them for forecasting. We just made training
samples and initial values of our model were the same with those of selected six
models. At an 8-month lead time, the correlation ability $T_C$ of our model for all
seasons combined was 0.613 (Fig. 10). In brief, the forecast ability of the ECMWF
model was slightly better than that of our model but the ability of the other 5 models
was worse than that of our model. While, in regard to the forecast length, the temporal
$T_C$ within 12 months of our model is greater than 0.6, which was superior
to the ECMWF model. In addition, the forecast results of the UCLA-TCD model and
the CPC CA model reduced quickly after 5-month lead times, so the forecast ability of
our model was more stable than them.

The root mean square error (RMSE) was also examined to assess the performance of discrimination and calibration. Barnston et al. (2012) believed that all seasonal RMSE values contributed equally to a seasonally combined RMSE. So we drew figure 42-11 to show seasonally combined RMSE.

From Fig. 14-0 and Fig. 421, we can see the highest correlation tend to have lower RMSE. So the RMSE of our model was slightly higher than that of ECMWF model, but it was much lower than those of the other 5 models. Figure 11 and Figure 12 is the average CPECTC and RMSE of the 240 experiments of compared with six mature models, covers a variety of different types of ENSO and different lead time. So those samples should be really representative.

6. Conclusions and discussion

6.1 Conclusions

A new forecasting model of the SST A field was proposed based on a dynamic system reconstruction idea and the principle of self-memorization. The approach of the present paper consisted of the following steps:

1. The SST field can be time (coefficients)-space (structure) deconstructed using the EOF method. Take $r$, $T$, SOI and EAWMI and consider them as trajectories of a set of four coupled quadratic differential equations based on the dynamic system reconstruction idea. The parameters of this dynamic model were estimated using a GA.

2. The forecast results of the dynamic model can be improved by the
self-memorization principle. The memory coefficients in the improved self-memorization model were obtained using the GA method.

(3) The long-term step-by-step forecast results and cross-validated retroactive hindcast results of time series $T_1$ and $T_2$ are all found to be good, with the correlation coefficient $CC$ of approximately 0.80 and a mean absolute percentage error the MAPE of less than 15%.

(4) The improved model was used to forecast the SSTA field. The forecasted SSTA fields of three types of events are accurate. Not only is the forecast shape similar to the actual field but also the contour lines are similar.

(5) The improved model was also used to forecast the ENSO index. The average correlation coefficient $TC$ of 60 examples within 12 months is 0.712, and the MAPE value is small, only 7.62%, which proves that the improved model has better forecasting results of the ENSO index. Although the forecast results of the model in the summer were worse than in the winter, the margin was not high, which means that the model can overcome the spring predictability barrier to some extent. Finally, compared with the six mature models, the new dynamical-statistical forecasting model has a scientific significance and practical value for the SST in the eastern equatorial Pacific and El Niño/La Niña event predictions.

6.2 Discussion

L’Heureux et al. (2013) reported that using different data sets and time periods, the 2nd EOF is not stable, being entirely due to the strong trend. So we need to do more experiments to prove that we choose the second mode of EOF to be appropriate.
and whether different time periods will make us forecast unstable or not. Our original

data is the monthly average SST data from January 1951 to Dec. 2010, which are 60

years. We will increase the length of the data for 20 years (Jan.1931 –Dec.2010), for

10 years (Jan.1941- Dec.2010) and decrease the length of the data for 10 years

(Jan.1961- Dec.2010), for 20 years (Jan.1971- Dec.2010). And then we use the same

method to reconstruct a model and forecast the ENSO index as section5.4. The

prediction results are shown in the table5.

From the table, we can see that in the 60 experiments, the prediction results of

the data period increased by 20 years are the best, and the prediction results of the

data period decreased by 20 years is the worst. This is because the more data we use,

the more information it contain. But from the table we can also see the difference

among forecast results of both TC and MAPE of five different sample data are less,

and no abnormal change suddenly worse or better appear. All these indicate that using

different data sets and time periods, even though may have a certain impact on the

pattern of the 2nd EOF, but the impact on our forecast is not great and it will not

make our forecast unstable.

Actually, how many variables and which variables are used in our model

become a key issue to be resolved. We are a complex four factor differential

equations coupling model. We are a complex coupled model of four factor differential

equations, so we are more concerned with the correlation between each other. The

correlation must be considered as an important criterion to select the factors, but in

order to further verify the correctness of the selection criterion, we have carried out
the prediction experiments (the 60 cross-validated retroactive hindcasts experiments of the ENSO index for all seasons combined at lead times of 8 months) of different variables.

We can see that for all the forecast results of the models of different variables, the prediction results of $\text{TT, SOI}$ is the best among those of the three factors and the prediction result of $\text{TT, SOI, EAWM}$ is the best among those of the four factors. But the prediction result of $\text{TT, SOI, EAWM}$ is best among all, which proves that our selection factors are correct. In our previous study (Hong et al., 2015), the model of the Western Pacific subtropical high was established by using the correlations as a criterion to select factors and their forecast results are also good. Now we use the correlations as a criterion to select factors is also in line with our previous research.

Because the formula of our model includes a linear combination of 4 variables ($\text{TT, SOI, EAWM}$), statistical forecasting requires independence between predictors. We can calculate the correlation coefficients between variables, as shown in Table 5. In fact, as Table 5 shows, the correlation coefficients between the factors were all less than 0.45, indicating the independence between factors. So this does not generate too much redundancy and can avoid an overfitting problem, which can destroy the stability of the model.

The definition of overfitting: The learned hypothesis may fit the training set very well, but fail to predict to new examples (fail to fit additional data or predict future observations reliably).

The potential for overfitting depends not only on the number of parameters and...
data but also the conformability of the model structure with the data shape, and the
magnitude of model error compared to the expected level of noise or error in the
data (Burnham and Anderson, 2002). So there are many reasons causing the overfitting
phenomenon. But this does not mean having many parameters relative to the number
of observations inevitably causes the overfitting problem (Golbraikh et al., 2003).
There is no evidence that more parameters will be certain to result in overfitting.
Based on the definition of overfitting and the previous studies (Golbraikh et al., 2003;
Everitt and Skrondal, 2010), we can judge whether a model is overfitting or not by the
accuracy of prediction results of independent samples (Golbraikh and Tropsha, 2002;
Qin and Li, 2006).

In the sample training, our model does not purposely pursue the high degree of
the training samples fitting and improve the effectiveness of the independent
generalization. In fact, in our paper the forecast results of the Cross-validated
retroactive hindcasts (section 5.2) and the independent samples validation (Table 3 and
Table 4) are both good. Especially, the independent samples validation of the ENSO
index as the Table 4, we have carried out the 240 independent sample validation
prediction of four seasons of different ENSO events and the coverage of independent
samples test is very wide. Moreover, compared with 6 mature prediction models, the
forecast results of our model are also good, which prove the overfitting problem does
not exist in our model. According to the previous literature (Islam and Sivakumar,
2002; Sivakumar et al., 2001), we can see that prediction principle and structure of the
phase space reconstruction (PSR) of dynamical system is not the same with the
traditional neural network and in the small sample situation the forecasting results of PSR model are better than those of the traditional neural network (Sivakumar et al., 2002), which can be verified in the independent sample test (table 3 and table 4). So according to the definition of overfitting, we can say the over fitting phenomenon does not exist in our model.

The introduction of self memorization essentially introduces a lot of new coefficients, which may cause an over fitting problem. Because we have selected a model of four variables, there is a total of 62 parameters. In order to avoid the over fitting problem, the sample sizes are more than 10% of the amount of parameters. So our sample size is greater than 620 data to avoid the overfitting problem. If we choose the model of three variables, the parameters in which will be less, the sample size in this situation can be less. But the forecast results may be a little worse, based on the analysis in section 2.3. So the length of training samples is related to the number of parameters of our model.

Also, we have tried to detrend our data before the model constructed. But we found the results didn’t change too much. That is mean our model is not very sensitive to climate change, so the detrended data has little effect for our model to improve the forecast effect.

Compared with the original model, why the improved model has good forecast results and can overcome the spring predictability barrier to some extent are as follow:

Recently, many studies have pointed out that spring is the most unstable season of the air-sea interaction and the error is likely to develop or grow in the spring, resulting in
the spring predictability barrier (Zhang et al, 2012; Philander et al., 1992). When the
original model uses the indexes in summer as the initial values to predict, the SOI
factor representing the air-sea interaction is most unstable in the spring and the
EMWMI factor does not have much influence on ENSO in summer, so the forecast
results using the indexes in summer as the initial values are certainly much worse than
those using the indexes in the winter as the initial values. That is why our original
model does not overcome the spring predictability barrier.

However, the introduction of the self-memorization dynamics principle can help
our model overcome the spring predictability barrier to some extent. Although the
lead time is still summer (such as JJA), the information of the initial value actually
contains the previous $p + 1$ month (in this case $p = 6$, which contains the information
of the previous seven months, including the information of $T_1, T_2$, SOI, EMWMI
factor in winter (January, February), spring (March, April, May) and summer (June
and July)). From the dynamical analysis, in this situation, the information and
interaction relationship of four factors have been a long period (from winter to
summer) accumulated, containing much air-sea interaction processes and winter
monsoon continued abnormal information, so the forecast results of our improved
model will be much better than the original model which simply uses only one initial
value. That is why the improved model overcomes the spring predictability barrier to
some extent.

The forecast results of our model are good, but it still has some problems:

1. The inclusion of these terms and the physical processes do these terms in
equation (2) represent are important, especially for the discussion of dynamical characteristics of the dynamical model. But now we are difficult to give a clear meaning. Now the main work of our paper is the prediction experiments of the model. For the reason of time and length, this paper mainly discusses the prediction results of the model. The physical processes do these terms represent and the discussion of the dynamical characteristics of the model will be the focus of our next work. Before this, we have also used the Takens’ delay embedding theorem to reconstruct the dynamical model of the Western Pacific subtropical high (WPSH). And Based on the reconstructed dynamical model, dynamical characteristics of WPSH are analyzed and an aberrance mechanism is developed, in which the external forcings resulting in the WPSH anomalies are explored, which have been published (Hong et al., 2016). We also study the bifurcation and catastrophe of the West Pacific subtropical high ridge index of a nonlinear model (Hong et al., 2017). Based on our previous method and work, our next work is to analyse the physical processes and the dynamical characteristics of the SST field.

Although the reason why the improved model has good forecast results has discussed in the section 6.2, the deep physical mechanisms that the proposed model has dealt with is not very clear, so its dynamical characteristics should be further analysed.

(2) The experiments in the present study have proven that the forecasting results of the improved model are good for large-scale systems, such as ENSO events, and the forecasting period has been extended. However, for small-scale systems, such as
Hurricanes, whether the forecast results could be improved using the present improved model needs to be further verified.

(3) Our paper focuses primarily on these defined indices with $\tau_i, r_i$ to reconstruct a prediction model. Maybe, we can select variables (predictor) based on EOF analysis and our model may be a more physically oriented model. Maybe we can learn from Yim et al. (2013; 2015) to draw correlation maps between these fields and the SSTA field and select the predictors from physical considerations. All these above questions require that a lot of experiments to be carried out.

These items will be our future work.

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APPENDIX A: THE PRINCIPLE OF DYNAMICAL MODEL RECONSTRUCTION

Suppose that the physical law of a nonlinear system going by over time can be expressed as the following difference form:
where \( f_i \) is the generalized nonlinear function of \( q_i, q_{i+1}, \ldots, q_{M} \). \( M \) is the length of observed data. \( f_i(q_i, q_{i+1}, \ldots, q_{M}) \) can be assumed to contain two parts: \( G_x \) representing the expanding items which contain variable \( q_i, p_{x_i} \) just representing the corresponding parameters which are real numbers \( i = 1, 2, \ldots, N, j = 1, 2, \ldots, M, k = 1, 2, \ldots, K \).

It can be supposed as follows:

\[
f_i(q_i, q_{i+1}, \ldots, q_M) = \sum_{k=1}^{K} G_{jk} p_k
\]  

\( D = GP \) is the matrix form of Eq.(A2), in which

\[
D = \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_M \end{bmatrix} = \begin{bmatrix} \frac{q_i^{12} - q_i^{1M}}{2M} \\ \frac{q_i^{22} - q_i^{2M}}{2M} \\ \vdots \\ \frac{q_i^{M2} - q_i^{MM}}{2M} \end{bmatrix}, \quad G = \begin{bmatrix} G_{11}, G_{12}, \ldots, G_{1k} \\ G_{21}, G_{22}, \ldots, G_{2k} \\ \vdots \\ G_{M1}, G_{M2}, \ldots, G_{Mk} \end{bmatrix}, \quad P = \begin{bmatrix} p_{11} \\ p_{12} \\ \vdots \\ p_{1k} \end{bmatrix}
\]  

Parameters of the above equation can be determined through inverting the observed data. Vector \( P \) which satisfies the above equation can be solved, based on a given vector \( D \). Assuming \( q \) is unknown, it is a nonlinear system. However, assuming \( P \) is unknown, it is a linear system.

With the restriction \( S = (D - GP)^T (D - GP) \) as a minimum, GA is introduced as an optimization solution search in the model parameters space.

Assuming that the parameters matrix \( P \) is the population (solutions), the

\( S = (D - GP)^T (D - GP) \) is an objective function, \( l_i = \frac{1}{S} \) is the value of individual fitness, and \( L = \sum_{i=1}^{n} l_i \) is the value of total fitness. The operating steps of GA include:

creation and coding of initial population (solutions), fitness calculation, the choice of
male parents, crossover and variation, etc. A detailed theoretical explanation can be
got from Wang (2001). The step length is 1 month during the calculation. After
optimization searches and genetic operations, the target value can be rapidly
converged on and each optimal parameter of the dynamical equations can be obtained.

Through the above approach, we can obtain parameters of a nonlinear
dynamical system, and reconstruct the nonlinear dynamical equations from observed
data.

APPENDIX B: THE MATHEMATICAL PRINCIPLE OF
SELF-MEMORIZATION DYNAMICS OF SYSTEMS

The dynamical equations of a system can be expressed as:

$$\frac{\partial x}{\partial t} = F_i(x, \lambda, t) \quad i = 1, 2, ..., J$$  \hspace{1cm} (B1)

where \( J \) is an integer, \( x \) is the \( i \)th variable of the system state, and \( \lambda \) is
the parameter. Equation (B1) represents the relationship between a source function
\( F \) and a local change of \( x \). Obviously, \( x \) is a scalar function with time \( t \) and
space \( r_0 \). A set of time \( T = [t_{i0}, ..., t_{ij}] \) can be considered, where \( t_{i0} \) is an initial
time. A set of space \( R = [r_{0i}, ..., r_{0j}] \) can be considered, where \( r_i \) is a spatial point.

An inner product in space \( L^2 : T \times R \) is defined by:

$$ (f, g) = \int_T^0 \int_R (f(\xi)g(\xi))d\xi, f, g \in L^2 $$  \hspace{1cm} (B2)

Accordingly, a norm can be defined as:

$$ \|f\| = \left[ \int_T^0 (f(\xi))^2 d\xi \right]^{\frac{1}{2}} $$
For a completion $L^2$, it can become a Hilbert space $H$. A generalized one in $H$ can be regarded as a solution of the multi-time model. By introducing a memorization function $\beta(r,t)$, we can obtain:

$$\int_0^t \beta(t) \frac{\partial x}{\partial \tau} d\tau = \int_0^t \beta(t) F(x,\tau) d\tau$$  \hspace{1cm} (B3)

where $r$ in $\beta(r,t)$ can be dropped through fixing on the spatial point $r_0$. Suppose that function $\beta(r,t)$ and variable $x$ etc. are all continuous, differentiable and integrable, an integration by the left parts of Eq. (B3) can be made as:

$$\int_0^t \beta(t) \frac{\partial x}{\partial \tau} d\tau = \beta(t)x(t) - \beta(t_0)x(t_0) - \int_0^t x(\tau) \beta'(\tau) d\tau$$  \hspace{1cm} (B4)

where $\beta'(t) = \frac{\partial \beta(t)}{\partial t}$. The mean value theorem can be introduced into the third term in Eq. (B4), the following equation can be obtained:

$$-\int_0^t x(\tau) \beta'(\tau) d\tau = -x^m(t_0)[\beta(t) - \beta(t_0)]$$  \hspace{1cm} (B5)

where $x^m(t_0) = x(t_m), t_0 < t_m < t$. Substituting Eq. (B4) and Eq. (B5) in Eq. (B3) and carrying out an algebraic operation, the following equation can be obtained:

$$x(t) = \frac{\beta(t_0)}{\beta(t)} x(t_0) + \frac{\beta(t) - \beta(t_0)}{\beta(t)} x^m(t_0) + \frac{1}{\beta(t)} \int_0^t \beta(\tau) F(x,\tau) d\tau$$  \hspace{1cm} (B6)

Because the $x$ value which is at initial time $t_0$ and middle time $t_m$, only on the fixed point $r_0$ itself, relates to the first term and the second term in Eq. (B6), they are be called as a self-memory term. Also, we can call the third term as an exogenous effect, i.e., which is contributed by other spatial points.

Similarly as Eq. (B4), for multi-time $t_i, \ i = -p, -p+1, ..., t_0, t$, it gives
\( \int_{t_{-p}}^{t} \beta(t) \frac{\partial x}{\partial \tau} \, d\tau + \int_{t_{-p}}^{t-p-1} \beta(t) \frac{\partial x}{\partial \tau} \, d\tau + \ldots + \int_{t_{0}}^{t} \beta(t) \frac{\partial x}{\partial \tau} \, d\tau = \int_{t_{-p}}^{t} \beta(t) F(x, \tau) \, d\tau. \)

After the same term \( \beta(t_i) x(t_i) i = -p + 1, -p + 2, \ldots, 0 \) was eliminated, we have

\[ \beta(t) x(t) - \beta(t_{-p}) x(t_{-p}) - \sum_{i=-p}^{0} \left[ \beta(t_{i+1}) - \beta(t_i) \right] x^{''}(t_i) - \int_{t_{-p}}^{t} \beta(t) F(x, \tau) \, d\tau = 0 \quad (B7) \]

As a matter of convenience, we set \( \beta_i = \beta(t_i), \beta_0 = \beta(t_0), x_i = x(t_i), x_0 = x(t_0) \); the following text uses similar notations. Then, Eq. (B7) can be expressed as:

\[ \beta_i x_i - \beta_{-p} x_{-p} - \sum_{i=-p}^{0} x_i'' \left( \beta_{i+1} - \beta_i \right) - \int_{t_{-p}}^{t} \beta(t) F(x, \tau) \, d\tau = 0 \quad (B8) \]

Setting \( x_{-p} = x^{''}_{-p-1}, \beta_{-p-1} = 0 \), the Eq. (B8) can be written as:

\[ x_i = \frac{1}{\beta_i} \sum_{i=-p}^{0} x_{i+1}'' \left( \beta_{i+1} - \beta_i \right) + \frac{1}{\beta_i} \int_{t_{-p}}^{t} \beta(t) F(x, \tau) \, d\tau = S_1 + S_2 \quad (B9) \]

\( S_1 \) is called as a self-memory term and \( S_2 \) is called as an exogenous effect term.

For the convenience of calculations, the above self-memorization equation can be discretized. The differential by difference and the summation can replace the integration in Eq. (B9), and the mean of two values which are at adjoining times; i.e.,

\[ x_i'' \approx \frac{1}{2} (x_{i+1} + x_i) = y_i \] can simply replace \( x_i'' \).

Taking an equal time interval \( \Delta t = t_{i+1} - t_i = 1 \) and incorporating \( \beta_i \) and \( \beta_i \), we can obtain a discretized self-memorization equation as follows:

\[ x_i = \sum_{i=-p-1}^{-1} \alpha_i y_i + \sum_{i=-p}^{0} \theta_i F(x, i) \quad (B10) \]

where \( F \) is the dynamic kernel of the self-memorization equation, \( \alpha_i = \frac{\beta_{i+1} - \beta_i}{\beta_i} \); \( \theta_i = \frac{\beta_i}{\beta_i} \).
Based on Eq. (B10), the above technique performed computations and the forecast can be called as a self-memorization principle.

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Fig. 3. The cross-validated retroactive hindcast results of the first time coefficient series (a) and the second time coefficient series (b) of the SSTA field by the original model.

Fig. 4. Long-term step-by-step forecast results of the first time coefficient series (a) and the second time coefficient series (b) of the SSTA field by the improved model.

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Fig. 8. The forecast SSTA field (a) and the actual SSTA field (b) of a neutral event (Nov. 2002).

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Fig. 10. The cumulative correlation coefficients (a) and cumulative mean absolute percentage error (b) changing with time of different lead times.

Fig. 11. Temporal correlation between model forecasts and observations for all seasons combined, as
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Fig. 1. RMSE in standardized units, as a function of lead time for all seasons combined. Each line highlights one model.

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Table 2. The correlation coefficient (CC) and Mean absolute percentage error (MAPE) of long-term fitting test when the retrospective order $p$ is different.

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Table 4. The TC and the MAPE between model forecasts and observations within 12 months for November–January, December–February, and January–March as lead time of winter and for May–July, June–August and July–September as lead time of summer.

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(a)
Fig. 4. Long-term step-by-step forecast results of the first time coefficient series $T_1 \mathcal{T}_T$ (a) and the second time coefficient series $T_2 \mathcal{T}_T$ (b) of the SSTA field by the improved model.
(a)
Fig. 5. The cross-validated retroactive hindcast results of the first time coefficient series $T_1$ (a) and the second time coefficient series $T_2$ (b) of the SSTA field by the improved model.
Fig. 6. The forecast SSTA field (a) and the actual SSTA field (b) of an El Niño event (Dec. 1997)
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Fig. 1. Temporal correlation between model forecasts and observations for all seasons combined, as a function of lead time. Each line highlights one model.
Fig. 1. RMSE in standardized units, as a function of lead time for all seasons combined. Each line highlights one model.
Table 1. The correlation analysis between the front two time series $\mathbf{TT}$ and nine impact factors

<table>
<thead>
<tr>
<th>factors</th>
<th>$u_1$</th>
<th>$u_2$</th>
<th>PNA</th>
<th>DMI</th>
<th>SOI</th>
<th>PDOI</th>
<th>EAWMI</th>
<th>OLR</th>
<th>SSH*</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_1$</td>
<td>0.3161</td>
<td>0.5684</td>
<td>0.4386</td>
<td>-0.3457</td>
<td>0.7734</td>
<td>0.4081</td>
<td>0.6284</td>
<td>0.3287</td>
<td>0.3363</td>
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<tr>
<td>$T_2$</td>
<td>0.2118</td>
<td>0.4181</td>
<td>0.2560</td>
<td>-0.2345</td>
<td>0.5232</td>
<td>0.3065</td>
<td>0.4825</td>
<td>0.1816</td>
<td>0.2169</td>
</tr>
</tbody>
</table>

Table 1. The forecast results of the models of different variables

<table>
<thead>
<tr>
<th>The model</th>
<th>The forecast skill of 60 cross-validated retroactive hindcast experiments of the ENSO index for all seasons combined at lead times of 8 months</th>
<th>the temporal correlation</th>
<th>the root mean square error</th>
</tr>
</thead>
<tbody>
<tr>
<td>One variable ($T_1$)</td>
<td>0.5051</td>
<td>0.8075</td>
<td></td>
</tr>
<tr>
<td>Two variables ($T_1, T_2$)</td>
<td>0.5613</td>
<td>0.7629</td>
<td></td>
</tr>
<tr>
<td>Three variables ($T_1, T_2, SOI$)</td>
<td>0.6027</td>
<td>0.7225</td>
<td></td>
</tr>
<tr>
<td>Four variables ($T_1, T_2, SOI, EAWMI$)</td>
<td>0.6344</td>
<td>0.6728</td>
<td></td>
</tr>
<tr>
<td>Five variables ($T_1, T_2, SOI, EAWMI, u$)</td>
<td>0.5923</td>
<td>0.7344</td>
<td></td>
</tr>
<tr>
<td>Six variables ($T_1, T_2, SOI, EAWMI, u, PNA$)</td>
<td>0.5528</td>
<td>0.7806</td>
<td></td>
</tr>
</tbody>
</table>
Table 2. The correlation coefficient (CC) and mean absolute percentage error (MAPE) of long-term fitting test when the retrospective order \( p \) is different

<table>
<thead>
<tr>
<th>( p )</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>The forecast results of long-term fitting test</td>
<td>CC</td>
<td>0.75</td>
<td>0.73</td>
<td>0.81</td>
<td>0.74</td>
<td>0.70</td>
<td>0.72</td>
</tr>
<tr>
<td>MAPE</td>
<td>18.42%</td>
<td>19.36%</td>
<td>14.56%</td>
<td>20.39%</td>
<td>25.31%</td>
<td>24.18%</td>
<td>27.33%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( p )</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>The forecast results of long-term fitting test</td>
<td>CC</td>
<td>0.68</td>
<td>0.70</td>
<td>0.65</td>
<td>0.62</td>
<td>0.60</td>
</tr>
<tr>
<td>MAPE</td>
<td>28.10%</td>
<td>26.58%</td>
<td>30.91%</td>
<td>33.14%</td>
<td>34.97%</td>
<td>33.56%</td>
</tr>
</tbody>
</table>
Table 3. The forecast results of $T_1$ and $T_2$ in different examples within 6 and 12 months

<table>
<thead>
<tr>
<th>Forecast events</th>
<th>The results within 6-months</th>
<th>The results within 12-months</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CC</td>
<td>MAPE</td>
</tr>
<tr>
<td>The average of 18 El Niño examples of $T_1$</td>
<td>0.824</td>
<td>8.45%</td>
</tr>
<tr>
<td>The average of 22 La Niña examples of $T_1$</td>
<td>0.846</td>
<td>7.68%</td>
</tr>
<tr>
<td>The average of 20 Neutral examples of $T_1$</td>
<td>0.885</td>
<td>6.23%</td>
</tr>
<tr>
<td>The average of total 60 examples of $T_1$</td>
<td>0.850</td>
<td>7.41%</td>
</tr>
<tr>
<td>The average of 18 El Niño examples of $T_2$</td>
<td>0.811</td>
<td>8.79%</td>
</tr>
<tr>
<td>The average of 22 La Niña examples of $T_2$</td>
<td>0.833</td>
<td>7.35%</td>
</tr>
<tr>
<td>The average of 20 Neutral examples of $T_2$</td>
<td>0.896</td>
<td>6.68%</td>
</tr>
</tbody>
</table>
Table 4. The TC and the MAPE between model forecasts and observations within 12 months for
lead time of spring, for May-July, June-August and July-Sep. as lead time of summer and for

<table>
<thead>
<tr>
<th>Forecast events</th>
<th>Lead time of all seasons combined</th>
<th>Lead time of spring</th>
<th>Lead time of autumn</th>
<th>Lead time of winter</th>
<th>Lead time of summer</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(MIJ-JJA-JAS)</td>
<td>(NDJ-DJF-JF)</td>
<td>(ASO-SON-ON)</td>
<td>(FMA-MAM-AM)</td>
<td>(J)</td>
</tr>
<tr>
<td>The average of</td>
<td>0.60</td>
<td>0.56</td>
<td>0.632</td>
<td>0.67</td>
<td>0.645</td>
</tr>
<tr>
<td>18 El Niño</td>
<td>9.70%</td>
<td>10.33%</td>
<td>8.85%</td>
<td>8.02%</td>
<td>11.6%</td>
</tr>
<tr>
<td>examples</td>
<td>4</td>
<td>9</td>
<td>7</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>The average of</td>
<td>0.62</td>
<td>0.58</td>
<td>0.645</td>
<td>0.69</td>
<td>0.579</td>
</tr>
<tr>
<td>22 La Niña</td>
<td>8.97%</td>
<td>9.82%</td>
<td>8.41%</td>
<td>7.83%</td>
<td>9.82%</td>
</tr>
<tr>
<td>examples</td>
<td>5</td>
<td>1</td>
<td>5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The average of total 60 examples of $T_2$: 0.842 7.64% 0.740 11.71%
### Table 4. Temporal correlation (CC) and the mean absolute percentage error (MAPE) between model forecasts and observations within 12 months for Nov–Jan, Dec–Feb, and Jan–Mar as lead time of winter and for May–July, June–August and July–Sep as lead time of summer.

<table>
<thead>
<tr>
<th>Forecast events</th>
<th>Lead time of all seasons combined</th>
<th>Lead time of summer (MJJ-JJA-JAS)</th>
<th>Lead time of winter (NDJ-DHF-JFM)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CC</td>
<td>MAPE (%)</td>
<td>CC</td>
</tr>
<tr>
<td>The average of 18 El Niño examples</td>
<td>0.604</td>
<td>9.70%</td>
<td>0.569</td>
</tr>
<tr>
<td>The average of 22 La Niña examples</td>
<td>0.623</td>
<td>8.92%</td>
<td>0.581</td>
</tr>
<tr>
<td>The average of 20 Neutral examples</td>
<td>0.798</td>
<td>5.96%</td>
<td>0.752</td>
</tr>
<tr>
<td>The average of total 60 examples</td>
<td>0.712</td>
<td>7.62%</td>
<td>0.633</td>
</tr>
</tbody>
</table>
The average of examples

<table>
<thead>
<tr>
<th>Periods</th>
<th>TC</th>
<th>MAP</th>
<th>TC</th>
<th>MAPE</th>
<th>TC</th>
<th>MAPE</th>
<th>TC</th>
<th>MAPE</th>
</tr>
</thead>
<tbody>
<tr>
<td>18 El Niño</td>
<td>0.69</td>
<td>9.70%</td>
<td>0.68</td>
<td>9.02%</td>
<td>0.642</td>
<td>9.35%</td>
<td>0.57</td>
<td>10.15%</td>
</tr>
<tr>
<td>22 La Niña</td>
<td>0.62</td>
<td>8.97%</td>
<td>0.70</td>
<td>8.33%</td>
<td>0.675</td>
<td>8.55%</td>
<td>0.58</td>
<td>9.42%</td>
</tr>
<tr>
<td>20 Neutral</td>
<td>0.79</td>
<td>5.96%</td>
<td>0.84</td>
<td>5.12%</td>
<td>0.821</td>
<td>5.56%</td>
<td>0.74</td>
<td>6.21%</td>
</tr>
<tr>
<td>total 60</td>
<td>0.71</td>
<td>7.62%</td>
<td>0.77</td>
<td>7.14%</td>
<td>0.740</td>
<td>7.38%</td>
<td>0.68</td>
<td>7.96%</td>
</tr>
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</table>

Table 5. The forecast results of the different data periods

<table>
<thead>
<tr>
<th>Periods</th>
<th>TC</th>
<th>MAP</th>
<th>TC</th>
<th>MAPE</th>
<th>TC</th>
<th>MAPE</th>
<th>TC</th>
<th>MAPE</th>
</tr>
</thead>
<tbody>
<tr>
<td>18 El Niño</td>
<td>0.69</td>
<td>9.70%</td>
<td>0.68</td>
<td>9.02%</td>
<td>0.642</td>
<td>9.35%</td>
<td>0.57</td>
<td>10.15%</td>
</tr>
<tr>
<td>22 La Niña</td>
<td>0.62</td>
<td>8.97%</td>
<td>0.70</td>
<td>8.33%</td>
<td>0.675</td>
<td>8.55%</td>
<td>0.58</td>
<td>9.42%</td>
</tr>
<tr>
<td>20 Neutral</td>
<td>0.79</td>
<td>5.96%</td>
<td>0.84</td>
<td>5.12%</td>
<td>0.821</td>
<td>5.56%</td>
<td>0.74</td>
<td>6.21%</td>
</tr>
<tr>
<td>total 60</td>
<td>0.71</td>
<td>7.62%</td>
<td>0.77</td>
<td>7.14%</td>
<td>0.740</td>
<td>7.38%</td>
<td>0.68</td>
<td>7.96%</td>
</tr>
</tbody>
</table>

Table 5. The correlation coefficients among four factors

<table>
<thead>
<tr>
<th>Correlation coefficients</th>
<th>SOI</th>
<th>EAWMI</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.77</td>
<td>7.14%</td>
<td>0.740</td>
</tr>
<tr>
<td></td>
<td>T</td>
<td>0.419</td>
</tr>
<tr>
<td>-------</td>
<td>------</td>
<td>-------</td>
</tr>
<tr>
<td>T</td>
<td>0.419</td>
<td></td>
</tr>
<tr>
<td>SOI</td>
<td>0.401</td>
<td>0.424</td>
</tr>
<tr>
<td>EAWMI</td>
<td>0.337</td>
<td>0.356</td>
</tr>
</tbody>
</table>

SOI    | 0.401 | 0.424 | 0.408 |