



Multivariate analysis of extreme storm surges in a semi-enclosed bay

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Abstract:

10 The prediction of extreme storm surges is a critical task for coastal area protection. This study examines extreme storm surges in Beibu Bay, a semi-enclosed bay in the South China Sea, and their joint probabilities. A method for the advanced prediction of the extreme storm surges is proposed using a multivariate extreme statistical method. We further present practical guidelines of the proposed multivariate analysis method, including guidelines for simulation. The simulation can be
15 extended to multidimensional data to simplify computation, so the proposed approach can be extended to use more points' data from the semi-enclosed bay for predicting extreme storm surges probabilities. A practical case study illustrates the application of the proposed techniques for extreme storm surges prediction. A comparison of the conditional probabilities obtained from observations and simulation data show that the proposed method is effective.

20 **Key Words:** Multivariate extreme analysis, Monte Carlo, SCS, Storm Surge

1. Introduction

The prediction of extreme storm surges due to typhoons is a critical problem for low-lying coastal areas. This problem is expected to increase because of climate change (Lowe et al., 2010; Weisse et al., 2012; Weisse et al., 2014; McInnes et al., 2003; Tebaldi et al., 2012; Arns et al., 2015).
25 Inundation caused by storm surges in the Chinese coastal region severely impacts and poses a continual threat to the activities and safety of the people living there. Between 1990 and 2010, storm surge disasters caused economic losses of approximately RMB 10.5 billion and 148 deaths, and affected 11.5 million annually. (Gao et al., 2014). Thus, storm surge prediction and rapid disaster



information dissemination are vital for facilitating evacuation and disaster mitigation in China.

30 This study focuses on extreme storm-surge probability prediction in Beibu Bay in the South
China Sea (SCS) using a multivariate extreme statistical method called multivariate generalized
Pareto distribution (MGPD). Recently, research on the application of multivariate extreme values
(EVs) has increased. In these studies, several possible probability distribution functions have been
used to characterize extreme sea level events: the Copula function (Salvadori et al., 2015; Corbella
35 and Stretch, 2012; Michele et al., 2007; Salvadori et al., 2013; Tao et al., 2013), multivariate EV
function (Morton and Bowers, 1996; Coles and Tawn, 1994; Bhunya, et al., 2011), and MGPD
function (which is a type of multivariate EV function) (Falk et al., 2004; Rootzén and Tajvidi, 2006).

In fact, multivariate EV analysis requires a reasonable extrapolation from observed states to
unobserved states. Therefore, in EV analysis, these distribution functions are not only fitted by
40 observed states, but they also predict unobserved states by reasonable extrapolation. The
multivariate EV and MGPD functions are derived from EV theory, a branch of probability theory
(Coles, 2001; Beirlant et al., 2005). However, only the MGPD function is the natural distribution of
the multivariate peak over threshold (MPOT) sampling method, which retains more extreme
information from the raw data than the annual maxima method (Zhang et al., 2000; Yap and Zhu,
45 2014; Menéndez and Woodworth, 2010). The annual maxima method often ignores multiple severe
storm waves that occur in the same year, which may be much larger than the annual largest waves
in many other years. Consequently, the annual maxima method may underestimate extreme
variables (You and Yin, 2006). Moreover, the uncertainty of the return level estimation by the peak
over threshold method is smaller than that by the annual maxima method under different sample
50 lengths (Yao et al., 2012).

In this paper, we do not specifically address MGPD theory; its details can be found in (Falk,
2004; Rootzén and Tajvidi, 2006; Beirlant et al., 2005; Tajvidi, 1996). We instead present our
developed MGPD procedures and show how the methodology can be exploited for the analysis of
extreme surges at two adjacent sites. The proposed theory and its statistical methodology are
55 presented in Section 2. In MGPD, determining the joint threshold and estimating the joint density
are critical tasks. These aspects are presented via an example in Sections 3 and 4, respectively.



Finally, MPOT advantages and application possibilities based on Monte Carlo simulation are outlined in the conclusion in Section 5.

2. Analysis Methodology

60 2.1 MGPD Theory

Multivariate EV theory has a broad range of applications (Coles and Tawn, 1994; Beirlant et al., 2005; Coles and Tawn, 1991). In this paper, we focus on the MGPD definition of Falk et al. (2004). The MGPD cumulative distribution function can be described as

$$\begin{aligned}
 W(X) &= 1 + \log(G(x_1, \dots, x_d)) \\
 &= 1 + \left(\sum_{i=1}^d x_i \right) D \left(\frac{x_1}{\sum_{i=1}^d x_i}, \dots, \frac{x_{d-1}}{\sum_{i=1}^d x_i} \right), \quad \log(G(x_1, \dots, x_d)) > -1
 \end{aligned} \tag{1}$$

65 where $(x_1, \dots, x_d) = x \in U$, where U is a neighborhood of zeros in the negative quadrant $(-\infty, 0)^d$, and $G(x_1, \dots, x_n)$ is a cumulative distribution function of the multivariate generalized EV distribution. In addition, D is the Pickands dependence function in the unit simplex $\overline{R_{d-1}}$ on the defined domain, i.e., $\overline{R_d} = \{x \in [0, \infty)^d \mid \sum_{i=1}^d x_i = 1\}$.

Function $W(X)$ can be deduced from $G(x_1, \dots, x_n)$, so the dependence relation form of
 70 $G(x_1, \dots, x_n)$ can greatly enrich the expression of $W(X)$ for various dependence relationships (Falk et al., 2004). Among them, $W(X)$ of the logistic type is widely used in hydrology, finance, and other fields because of the favorable statistical properties of the following Pickands dependence function:

$$D_r(t_1, \dots, t_{d-1}) = \left(\sum_{i=1}^{d-1} t_i^r + \left(1 - \sum_{i=1}^{d-1} t_i \right)^r \right)^{1/r}, \tag{2}$$

which means the MGPD cumulative distribution function can be rewritten as

$$75 \quad W_r(x) = 1 - \left(\sum_{i=1}^d (-x_i)^r \right)^{1/r} = 1 - \|x\|_r, \tag{3}$$

where r is the correlation parameter of the dependence function and $r > 1$. Further, x_i in the interval



$(-1, 0)$ denotes standardized variables. The bivariate logistic generalized Pareto distribution density function is given by

$$w_r(x, y) = \frac{\partial W}{\partial x \partial y} = (r-1)(xy)^{r-1} [(-x)^r + (-y)^r]^{1/r-2} \quad x < 0, y < 0. \quad (4)$$

80 Correlation parameter r can be evaluated using a step-by-step method, i.e., we first evaluate two marginal distributions and then evaluate r within MGPD including the marginal distributions. Alternatively, r can be evaluated using a global method in which r is directly estimated using the maximum likelihood for density function w . The global method more reliably evaluates the results because of its final function form; however, its evaluation processes are more complex. The
 85 maximum-likelihood function is

$$L(r) = \sum_{i=1}^n \ln(w_r(x_i, y_i)). \quad (5)$$

2.2 Simulation method

The Monte Carlo simulation method for a multivariate distribution is complex owing to the generation of multivariate random vectors. In general, the variables firstly are transformed into
 90 independent variables, for which random vectors are generated for each variable. The final random vectors of the multivariate distribution are obtained by an inverse transformation. The MGPD simulation method was suggested in Michel (2007).

To more effectively demonstrate the MGPD simulation method, we use T_p to transform vector (x_1, \dots, x_d) into polar coordinates as follows:

$$95 \quad T_p(x_1, \dots, x_d) = \left(\frac{x_1}{x_1 + \dots + x_d}, \dots, \frac{x_{d-1}}{x_1 + \dots + x_d}, x_1 + \dots + x_d \right) = (z_1, \dots, z_{d-1}, c), \quad (6)$$

where $c = x_1 + \dots + x_d$ and $\mathbf{Z} = (x_1/c, \dots, x_{d-1}/c)$ correspond to the radial and angular components, respectively, called Pickands polar coordinates.

In the Pickands polar coordinate system, $W(X)$ represents different properties. Assume that (X_1, \dots, X_d) follows MGPD $W(X)$ and the d -th differential of its Pickands dependence
 100 function D exists. We define the Pickands density $\phi(z, c)$ as follows (Falk et al., 2004):



$$\phi(z, c) = |c|^{d-1} \left(\frac{\partial^d}{\partial x_1 \cdots \partial x_d} W \right) T_p^{-1}(z, c) \quad (7)$$

Then ϕ depends only on z and, therefore, we put $\phi(z) = \phi(z, c)$. Under $\mu = \int_{R_{d-1}} \phi(z) dz > 0$ and constant $c_0 < 0$ exists and is close to zero, we can prove that c is a uniform distribution $P(C > c) = \mu|c|$ on $(c_0, 0)$. For details see Falk et al. (2004). So MGPD simulation method (details in Michel, 2007): 1) generates random numbers uniformly distributed on unit simplex $\overline{R_{d-1}}$, 2) generates random vector (z_1, \dots, z_d) using density function $f(z) = \frac{\phi(z)}{\mu}$ of $z = (z_1, \dots, z_{d-1})$ in Pickands polar coordinates and an acceptance-rejection method, 3) generates random numbers uniformly distributed on $(c_0, 0)$, and 4) calculates vector $(cz_1, \dots, cz_{d-1}, c - c \sum_{i=1}^{d-1} z_i)$, which is a random vector that satisfies the multivariate over threshold distribution. Constant c_0 is the joint threshold in the MGPD method, determined in this study using the principle proposed in (Coles and Tawn, 1994).

3. Data and Declustering

3.1 Data

The data used in this study were provided by the Joint Archive for Sea Level (JASL) of the University of Hawaii Sea Level Center (<http://uhslc.soest.hawaii.edu/home>). The data consist of simultaneous hourly sea-level observations at Beihai and Dongfang stations, which are located on the Beibu Bay coast in SCS (Fig. 1) and were collected from June 1975 to December 1997. The data can be used for the analysis of extreme surges because hourly sampling sufficiently captures high water levels. The missing values for Beihai correspond to only 0.023% of the data, while for Dongfang, 0.173% of the data was eliminated from the sample because of gauge malfunctions or other issues. 201,578 and 201,275 hourly values for Beihai and Dongfang, respectively, were to be processed, and the available data was judged sufficient to perform EV analysis of storm surges.

3.2 Data Analysis and Declustering

Beihai City, located in the south of Guangxi province of China, is a beach city on the coast of



125 Beibu Bay, which is a shallow, semi-closed bay. Owing to Beibu Bay's special geomorphologies,
its typhoon surges are violent and can cause flooding in the city. The surge levels at the site are
defined as the residuals after the astronomically induced tidal components have been removed from
the sea-level observations. The tidal component is cyclical and does not satisfy the basic hypothesis
of a random variable. Godin's method (Godin, 1972) was used for tidal analysis.

130 The first stage in EV analysis is declustering, specifically, identifying a set of independent
events. Declustering is performed to make adjacent elements of a sample that consists of the maxima
of events independent of each other. s. Our approach is to use a moving window of sufficient width
to ensure that the extremes of each variable from a single 'meteorological event' are certain to fall
within a single window (Coles and Tawn, 1994). Declustering techniques by Morton and Bowers
135 (1996) were used. The features of storms and storm surges differ in each place. In Beibu Bay, the
main meteorological event, can generate extreme wave height, is a typhoon. The declustering of a
sequence of surges is illustrated in Fig. 2. The duration of a typhoon surge in the Beibu Gulf is
approximately 100 h. The components of each vector are the maximum surge at each site over a
100-h event. The peak events for the vectors in the cluster occurred at different times. The peak
140 surges arrived at Dongfang 3-5 h before they arrived at Beihai, as shown in Fig. 2.

The main purpose of this study was to analyze the relationship between the extreme surges at
both sites, and then predict extreme storm surge probabilities at one site. Therefore, the declustering
method was judged to be appropriate because all the main peaks were included in these clusters.
The 100-h cluster interval enabled the surge peaks at both sites from the same typhoon to be in the
145 same cluster. According to the above principles, the total number of independent events N was 2,016.

3.3 Constructing Conditional Probability Functions

Extreme surges in the Beibu Gulf is are predominantly caused by typhoons from lower-latitude
areas in the SCS. Typhoons typically move through the Beibu Gulf from south to north, with a small
number of them moving from east to west. An extreme surge at Dongfang, which is to the south of
150 Beihai, can serve as an early warning signal for Beihai. Multivariate EV analysis can be used to
provide such a warning.

To analyze the joint probability of extreme surges in Beihai and Dongfang, conditional



probability (*CP*) distributions can be used (eq. 9). The value of *CP* represents the probability of encounters between extreme surges. The bivariate Pareto distribution function $W(x, y)$ is

$$155 \quad W(x, y) = Pr(X < x, Y < y) \quad (8),$$

where x and y represent surges m in Beihai and Dongfang, respectively. Distributions $W_x(x)$ and $W_y(y)$ are the marginal distributions of x and y , respectively. The *CP* distributions are as follows:

$$\begin{aligned} CP1: Pr(X \geq x | Y \geq y) &= \frac{Pr(X \geq x, Y \geq y)}{Pr(Y \geq y)} = \frac{1 - W_x(x) - W_y(y) + W(x, y)}{1 - W_y(y)} \\ CP2: Pr(X \leq x | Y \geq y) &= \frac{Pr(X \leq x, Y \geq y)}{Pr(Y \geq y)} = \frac{W_x(x) - W(x, y)}{1 - W_y(y)} \\ CP3: Pr(X \geq x | Y \leq y) &= \frac{Pr(X \geq x, Y \leq y)}{Pr(Y \leq y)} = \frac{W_y(y) - W(x, y)}{W_y(y)} \\ CP4: Pr(X \leq x | Y \leq y) &= \frac{Pr(X \leq x, Y \leq y)}{Pr(Y \leq y)} = \frac{W(x, y)}{W_y(y)} \end{aligned} \quad (9).$$

The other four *CP* distributions (*Y* conditional on *X*) can be deduced by swapping two variables.

4. Extreme-value analysis

160 In this section, we focus on problems in extreme surge prediction that can be solved using the statistical methodologies of EV analysis. These problems include joint threshold analysis, stochastic simulation, and the statistics of multivariate extreme surges. Finally, the interpretation of the statistical results for the extreme surges at the two locations is briefly discussed.

4.1 Marginal Transformation and Joint Threshold

165 Generalized Extreme Value Distribution (GEVD) includes EV I II III type distribution and can describe accurately more EV events than any single component. So we choose that the marginal distributions of the 2,016 independent events could be described by the following GEVD:

$$F(x) = P(X < x) = \exp \left\{ - \left[1 - \xi \left(\frac{x - \mu}{\sigma} \right) \right]^{1/\xi} \right\}, \quad \xi \neq 0 \quad (10),$$

170 where ξ , σ , and μ are three GEVD parameters. They are estimated using maximum likelihood, which was suggested in Coles (2001) and Beirlant et al. (2005). Fig. 3 shows the probability plots (including the 95% confidence intervals) of the marginal distributions before MGPD is fit and



the estimations of parameters are shown in Tab. 1.

As described in Section 2.1, the MGPD variables must be close to zero in the negative quadrant. The margins of a bivariate distribution can be transformed into uniform ones as suggested by Michel
 175 (2007). To standardize the margins, the marginal MGPD distribution must be a negative exponential distribution. Using the Taylor expansion, we can transform the data onto $(-1, 0)$ by computing

$$y = \log F(x_i) = \log(1 + F(x_i) - 1) \approx F(x_i) - 1 \quad (11),$$

where $F(x_i)$ is the GEVD of index i ($i = 1, 2$), indicating Beihai or Dongfang. In Eq. (11), $F(x_i)$ is close to one because this study focuses on extreme observations.

180 Many dependence models between extreme variables such as logistic, bilogistic, and Dirichlet models have been suggested. However, the choice of dependence model is not usually critical to the accuracy of the final model (Morton and Bowers, 1996). Therefore, a simple bivariate logistic generalized Pareto distribution was selected. The MGPD model of this paper is based on a multivariate EV distribution. Its joint threshold can be calculated by the method from Coles and
 185 Tawn (1994). The joint threshold is $c_0 = -0.28$, and there are 218 combinations of Beihai and Dongfang with $c > c_0$. Fig. 4(a) shows the samples that are over the threshold value. In the left subfigure, $c_0 = -0.28$ is a curve, and all observations on the right side of this curve are greater than c_0 . The converted data is shown in the right subfigure, where $c_0 = -0.28$ is a line.

The correlation parameter r of the dependence function is estimated using maximum likelihood
 190 and is $r = 2.15$. Using the obtained estimates for all parameters, the joint extreme probability distribution function is illustrated in Fig. 4(b).

4.2 Comparison of Stochastic Simulation Results

Using the simulation method in Section 2.2, we generated a very large sample of bivariate extreme storm surges. In this section, we compare the CP results obtained by simulation and direct
 195 calculation. Fig. 5 shows the results of the stochastic simulation for $N = 10,000$ and $100,000$. The simulation results are in basic conformity with the observations, which shows that the MGPD simulation method is effective. The scatter diagrams directly show the simulation result; however, they require further quantitative analysis to objectively show their differences.

We use two CPs: $CPI P(X > x / Y > y)$ and $CP4 P(X < x / Y < y)$. Here, CPI is the probability



200 that the surge in Beihai exceeds x conditional on the surge in Dongfang exceeding y , and $CP4$ is the probability that the surge in Beihai does not exceed x conditional on the surge in Dongfang not exceeding y . Fig. 6 shows that the relative difference in value between the simulation and direct calculation depends on the number of simulations N . The subplots in Fig. 6 clearly show that the relative difference is reduced as N increases. When N reaches 1.5×10^6 , the maximum relative error
205 between the simulated and calculated results is 8.61%, which we consider satisfactory.

In addition, we conducted runtime experiments in which 1×10^4 , 5×10^4 , 10×10^4 , 1×10^6 , and 1.5×10^6 random vectors were generated on a desktop PC with an Intel Core i7 3.4 GHz processor. The runtimes were 3, 24, 82, 9,649, and 22,106 s.

To estimate the M -year surge of Beihai and Dongfang by means of a univariate analysis, the
210 Poisson-Gumbel distribution was used (Quek and Cheong, 1992; Naffa et al., 1991), which can be described by

$$F(x) = P(X < x) = e^{-\lambda[1-G(x)]} \quad (12),$$

where $G(x)$ is the Gumbel distribution.

Tables 2 and 3 show the values of CPI and $CP4$ based on the results from 1.5×10^6 simulations.
215 The tables list the calculated and stochastic simulation results for five CPI and $CP4$ groups for different combinations of M -year surges at Dongfang and Beihai. The two results are very similar. For instance, the directly calculated $P(X > x_{50} / Y > y_{10})$ is 12.94% and its simulation result is 14.06%. Their relative error is 8.61%, which is the maximum relative error for CPI . Moreover, the directly calculated $P(X < x_5 / Y < y_{50})$ is 99.35% and its simulation result is 94.13%. Their relative error is
220 5.25%, which is the maximum relative error for $CP4$.

4.3 Prediction of Extreme Surges

The relationship between extreme surges at Beihai and Dongfang can be analyzed using the CP . Because the peak surges at Dongfang occur earlier than those at Beihai, we use $CPI: P(X > x / Y > y)$ to warn of an extreme surge at Beihai. Tab. 2 shows that, when a 50-year surge appears at
225 Dongfang, the probability of a greater than 50-year surge occurring at Beihai is 94.55%.

Using long-term surge records, we can determine the relationship between extreme surges at



Beihai and Dongfang. Additionally, because of the special geographical relationship between the two locations, a peak surge at Dongfang is a signal that predicts a peak surge at Beihai. We can therefore predict the probability of different surges at Beihai and then take preemptive measures to reduce or eliminate the associated damages.

5. Conclusion and Discussion

5.1 Conclusion

The primary aim of this study is to illustrate how recent MGPD developments can be applied to marine disaster prediction. We not only developed a process for determining a joint threshold and simulation, but we conducted an analysis for extreme surge warnings using MGPD. MGPD is the natural distribution of the MPOT method, which can cull extreme information from raw data. A model based on multivariate EV theory and the intrinsic properties of EVs were also considered. A Monte Carlo MGPD simulation was used to determine the conditional probability of two extreme surges, and the proposed approach was judged to give satisfactory results by comparing the relative differences in conditional probabilities obtained from observations and simulation data.

5.2 New Possibilities Based on Monte Carlo Simulation

Extreme surge warnings would be more reliable if the relationships of extreme surges at three or more sites could be established. Using more related (affected by the same typhoon) sites' storm surge real time data for storm surge warnings of Beihai city, the forecast result may be more believable by the statistical method presented in the paper. In this paper, the theory of MGPD and its simulation was derived for multidimensional variables. The methodology could be extrapolated to higher dimensional space. Thus, difficulty of solving a procedure for MGPD cannot be exacerbated by the high dimensionality of the variables. A potentially more effective warning approach could be based on a Monte Carlo simulation. Once long-term (e.g., thousands of years) sea state data has been simulated, several ocean environmental factors can be quickly assessed by the law of large numbers.



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255 References

- Arns, A., Wahl, T., Dangendorf, S., & Jensen, J.: The impact of sea level rise on storm surge water levels in the northern part of the German Bight, *Coastal Engineering*, 96, 118-131, 2015.
- Beirlant, J., Segers, J., Teugels, J.: *Statistics of Extremes, Theory and Applications*. Chichester: Wiley, sec. 5.1, 2005.
- 260 Bhunya, P. K., Singh, R.D., Berndtsson, R., Panda, S.N.: Flood analysis using generalized logistic models in partial duration series, *J. Hydrol.*, 420–421(7):59–71, 2011.
- Coles, S. G., Tawn, J. A.: Modelling extreme multivariate events, *Journal of the Royal Statistical Society*, B53:377–392, 1991.
- Coles S G, Tawn J A.: Statistical methods for multivariate extremes: an application to structural design, *Appl*
265 *Stat.*, 43:1–48, 1994.
- Coles, S. G.: *An Introduction to Statistical Modeling of Extreme Values*, London: Springer, sec. 3.3, 2001.
- Corbella, S., Stretch, D. D.: Multivariate return periods of sea storms for coastal erosion risk assessment, *Nat. Hazards Earth Syst. Sci.*, 12:2699–2708. <http://dx.doi.org/10.5194/nhess-12-2699-2012>, 2012.
- Falk, M., Hüslér, J., Reiss, R. D.: *Laws of Small Numbers: Extreme and Rare Events*, 2nd ed. Basel: Birkhäuser,
270 2004.
- Gao, Y., Wang, H., Liu, G. M., Sun, X. Y., Fei, X. Y., Wang, P. T., Lv, T. T., Xue, Z. S., and He, Y. W.: Risk assessment of tropical storm surges for coastal regions of China, *J. Geophys. Res. Atmos.*, 119:5364–5374, doi:10.1002/2013JD021268, 2014.
- Godin, G.: *The Analysis of Tides*, Toronto: University of Toronto Press, 1972.
- 275 Lowe, J. A., Woodworth, P. L., Tom, K., et al.: Past and future changes in extreme sea levels and waves. *Understanding Sea-Level Rise and Variability*, 3:26–375, 2010.
- McInnes, K. L., Walsh, K. J. E., Hubbert, G. D., et al: Impact of sea-level rise and storm surges on a coastal



- community, *Nat Hazards*, 30(2):187–207, doi:10.1023/A:1026118417752, 2003.
- Menéndez, M., Woodworth P. L.: Changes in extreme high water levels based on a quasi-global tide-gauge data
280 set. *J. Geophys. Res.*, 115(C10):234–244. doi:10.1029/2009JC005997, 2010.
- Michele, C. D., Salvadori, G., Passoni, G., & Vezzoli, R.: A multivariate model of sea storms using
copulas, *Coastal Engineering*, 54(10), 734–751, 2007.
- Michel, R. (2007). Simulation of certain multivariate generalized pareto distributions, *Extremes*, 10(3), 83–107.
- Morton I. D., Bowers J.: Extreme value analysis in a multivariate offshore environment, *Appl Ocean Res*,
285 18:303–317, 1996.
- Naffa M G, Fanos AM, Elganiny M Y.: Characteristics of waves off the Mediterranean Coast of Egypt, *J Coastal
Res*, 7: 38–49, 1991.
- Quek S T, Cheong H F.: Prediction of extreme gusts accounting for seasonal effects, *J Struct Safety*, 11(2): 121–
129, 1992.
- 290 Rootzén, H., Tajvidi, N.: Multivariate generalized Pareto distributions, *Bernoulli*, 12:917–930, 2006.
- Salvadori, G., Durante, F., Tomasicchio, G. R., et al.: Practical guidelines for the multivariate assessment of the
structural risk in coastal and off-shore engineering, *Coast. Eng.*, 95:77–83, 2015.
- Salvadori, G., Tomasicchio, G.R., D'Alessandro, F.: Multivariate approach to design coastal and off-shore
structures, *J. Coast. Res.*, 65:386–391, <http://dx.doi.org/10.2112/SI65-066.1> (Special Issue), 2013.
- 295 Tajvidi, N.: Characterization and Some Statistical Aspects of Univariate and Multivariate Generalized Pareto
Distributions, Department of Mathematics, Chalmers Tekniska Hogskola Goteborg,
<http://www.math.chalmers.se/~nader/thesis.ps> 1996, 1996.
- Tao, S., Dong, S., Wang, N., Guedes Soares, C.: Estimating storm surge intensity with Poisson bivariate
maximum entropy distributions based on copulas, *Nat. Hazards*, 68:791–807, [http://dx.doi.org/10.1007/s11069-013-
0654-6](http://dx.doi.org/10.1007/s11069-013-
300 0654-6), 2013.
- Tebaldi, C., Strauss, B. H., and Zervas, C. E.: Modelling sea level rise impacts on storm surges along US coasts,
Environ Res Lett, 7(1):14032–14042, 2012.
- Weisse, R., von Storch, H., Niemeier, H., Knaack, H.: Changing North Sea storm surge climate: an increasing
hazard, *Ocean Coast. Manag.*, 68:58–68, <http://dx.doi.org/10.1016/j.ocecoaman.2011.09.005>, 2012.
- 305 Weisse, R., Bellafiore, D., Menéndez, M., Méndez, F., Nicholls, R. J., Umgiesser, G., Willems, P.: Changing



extreme sea levels along European coasts, *Coast Eng.*, 87:4–14, 2014.

Yao, L., Zhu, L.: A new model of peaks over threshold for multivariate extremes, *China Ocean Eng.*, 28(6):761–776, 2014.

You, Z.-J., Yin, B.S.: Estimation of extreme coastal wave heights from time series of wave data, *China Ocean Eng.*, 20(2): 225–241, 2006.

Yao, L., Zhu, L., Hu, J.: A new method for determining threshold in using PGCEVD to calculate return values of typhoon wave height, *China Ocean Eng.*, 26(2): 251–260, 2012.

Zhang, K., Douglas B. C., Leatherman, S. P.: Twentieth-century storm activity along the U.S. east coast, *J. Clim.*, 13:1748–1761, 2000.



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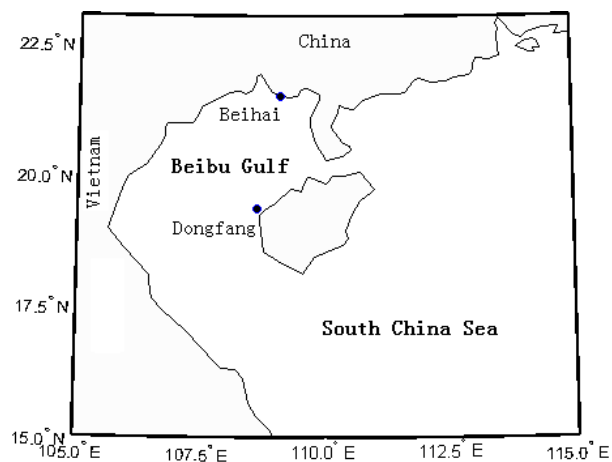
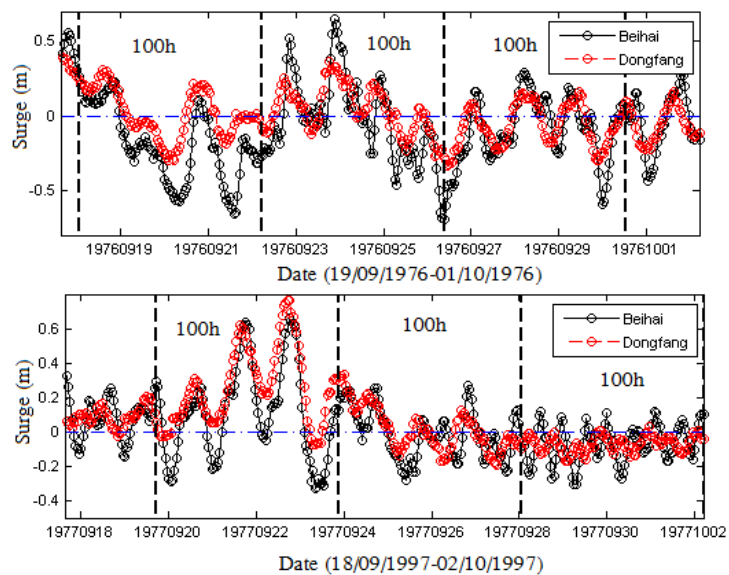


Fig. 1 Location of the two stations in Beibu Bay, SCS



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Fig. 2 Declustered surges

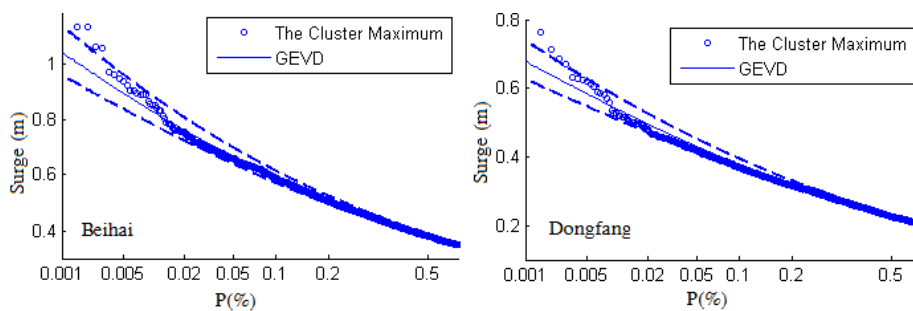


Fig. 3 Fitness testing of the marginal distribution

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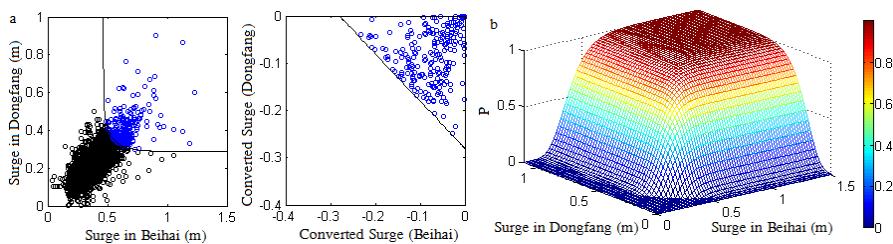


Fig. 4 (a) Dongfang and Beihai observations over the threshold value and (b) joint distribution of EVs for

Dongfang and Beihai

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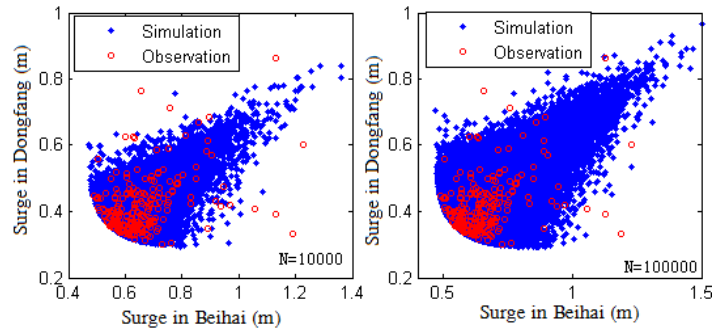


Fig. 5 Values over the threshold and stochastic simulation data

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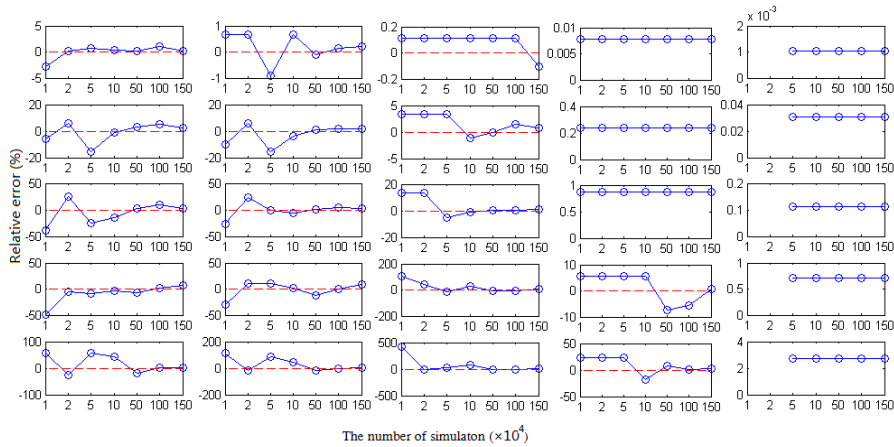


Fig. 6 Variance of the relative error under different numbers of simulation N for CPI . The conditional

probabilities for each subplot are shown in Tables 2

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Table 1 Parameters of the marginal distribution

	ξ	σ	μ
Beihai	0.3376	0.1187	-0.0465
Dongfang	0.1933	0.0890	-0.0720

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Table 2 Comparison of the results for *CPI* (%)

with the results from 1.5×10^6 simulations

RP (year)		5		10		20		50		100	
RP	D(m)	0.57		0.77		0.84		0.93		0.99	
	B(m)	c	s	c	s	c	s	c	s	c	s
5	0.87	62.52	62.59	99.33	99.54	99.89	99.79	99.99	100.00	100.00	100.00
10	1.13	5.20	5.34	81.00	82.52	96.67	97.44	99.76	100.00	99.97	100.00
20	1.22	1.70	1.74	49.38	50.71	88.05	88.70	99.13	100.00	99.89	100.00
50	1.34	0.34	0.37	12.94	14.06	47.95	49.68	94.55	95.12	99.29	100.00
100	1.42	0.11	0.11	4.29	4.47	19.09	19.40	80.75	82.93	97.37	100.00

RP: return period, a: analytic solution, s: simulation results, D: Dongfang, B: Beihai. (the same below)

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Table 3 Comparison of the results for *CP4* (%)

with the results from 1.5×10^6 simulations

RP (year)		5		10		20		50		100	
RP	D(m)	0.57		0.77		0.84		0.93		0.99	
	B(m)	c	s	c	s	c	s	c	s	c	s
5	0.87	99.75	97.61	99.36	94.26	99.35	94.15	99.35	94.13	99.35	94.12
10	1.13	100.0	99.99	99.98	99.81	99.97	99.72	99.97	99.69	99.97	99.69
20	1.22	100.0	100.0	100.0	99.97	99.99	99.93	99.99	99.90	99.99	99.90
50	1.34	100.0	100.0	100.0	100.0	100.0	99.99	100.0	99.98	100.0	99.98
100	1.42	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	99.99