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The timescale and extent of thermal expansion of the oceans due to climate change

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Abstract

With recently improved instrumental accuracy, the change in the heat content of the oceans and the corresponding contribution to the change of the sea level can be determined from in situ measurements of temperature variation with depth. Nevertheless, it would be favourable if the same changes could be evaluated from just the sea surface temperatures because the record could then be extended into the past and projected into the future. We show here that the average change in the heat content of the oceans and the corresponding contribution to a change in the sea level can be evaluated from the past sea surface temperatures. The calculation is based on the time-dependent diffusion equation with constant upwelling velocity and has no adjustable parameters. In the steady-state limit it recovers the well-known profile of the potential temperature variation as a function of depth.

The results are in good agreement with the estimates obtained from the in situ data, even though most of the warming occurs in the upper 1000 m. The method allows us to obtain relevant timescales and average temperature profiles. The evaluation of the thermosteric sea level change is extended back to the beginning of accurate sea surface temperature records in 1880. The changes in sea surface temperature from 1880 until the present time led to a thermosteric sea level rise of 3 cm and to a commitment for a future rise of 5 cm.

1 Introduction

The temperatures of the ocean interior and hence the global thermosteric sea level reflect the variations of the sea surface temperatures up to the present time. Improving the understanding of this unknown functional relationship should provide insight into the mechanism of ocean warming resulting from climate change and increase the accuracy of sea level predictions in future scenarios.

The details of the change in ocean heat content and the corresponding thermosteric

sea level increase depend on the global sea surface temperatures as a function of position and time (Domingues et al., 2008, Levitus et al., 2009). Nevertheless, simpler models that use spatial averaging over the whole or parts of the global ocean have in the past been successful in interpreting the observations. A famous example is the analysis of potential temperature and salinity variation as a function of depth (Munk, 1966) based on the steady state balance between the turbulent diffusion and upwelling. More recently, interest in simple globally averaged models was revived by Rahmstorf (2008) who used simple, semi-empirical modeling to describe the average sea level change. His work provided significant insight into the nature of sea level rise without a need to specify details of the processes involved. Rahmstorf assumed that recent changes have been so rapid that the rate of change of the sea level is proportional to the difference between the instantaneous temperature and a single (hypothetical) earlier temperature when the climate and the state of the oceans were in equilibrium. The assumption led to a reasonable but not detailed agreement between integrated temperature data and the sea level change.

Here we explore the time dependence of the average ocean warming using the well-known mixing equation combining vertical diffusion and advection (see e.g. Stewart, 2009)

$$\frac{\partial \psi(z,t)}{\partial t} = A_z \frac{\partial^2 \psi(z,t)}{\partial z^2} - W \frac{\partial \psi(z,t)}{\partial z}, \tag{1}$$

where $\psi(z,t)$ is the potential temperature or tracer density, A_z the vertical eddy diffusivity and W the mean vertical velocity. Munk (1966) reviewed the data and determined the characteristic length A_z/W from a fit to the thermocline in the Pacific between 1 km and 4 km depth and A_z/W^2 from the vertical distribution of ^{14}C. The resulting values $A_z \approx 1.3\times10^{-4}$ m^2/s and $W \approx 1.2$ cm/day are used throughout this work.

Most of the changes in the ocean heat content resulting from climate change are presently occurring in the top 1000 m. If the simple advection-diffusion model of Eq. (1) were valid in this region, changes due to the surface warming could be calculated

independently of the established steady-state thermocline because Eq. (1) is linear and superposition holds. In the subsequent sections we show that using the advection-diffusion model with Munk's average values for A_z and W to calculate the average ocean heat content and sea level change produces accurate results. Other than the arbitrary choice of the reference zero value for heat and sea level change, we do not need any adjustable parameters. The applicability of the advection-diffusion model with Munk's values for A_z and W is thus extended to the calculation of average global temporal variations, including those occurring in the upper ocean.

In the next section we start the analysis with a discussion of the response to a step change in surface temperature. The solution is required in the subsequent section, which uses sea surface temperatures as measured from 1880 to calculate the change in ocean heat content and thermosteric sea level. The approach is validated by the very good agreement with the findings based on in situ data. The report ends with a brief discussion, including the evaluation of the present-day commitment to future steric sea level increase.

2 Response to a step change in sea surface temperature

Properties of the solution for a step change in surface temperature are interesting because they indicate the time scales of the problem and the ultimate steric sea level change for each degree of surface warming. Let us assume the ocean is in the natural steady state temperature distribution and make a step change in surface temperature at the time $t = 0$, so that $\psi(z, t)=0$ for $t < 0$ and $\psi(0,t)=1$ for $t > 0$. The depth z is measured from the surface in downward direction and the velocity W is hence negative. The solution of Eq. (1) for this boundary condition is found using Laplace transforms (Ogata, 1970; Lee, 1999)

$$\psi(z,t) = \frac{1}{2}\exp\frac{Wz}{2A_z}\left(\exp\frac{-Wz}{2A_z} erfc\frac{z-Wt}{2\sqrt{A_z t}} + \exp\frac{Wz}{2A_z} erfc\frac{z+Wt}{2\sqrt{A_z t}}\right), \tag{2}$$

where *erfc* is the complementary error function. In the steady state limit $t \to \infty$, Eq. (1) reduces to the Munk profile $\psi(z,\infty) = \exp(-Wz/A_z)$. The boundary condition at the bottom of the ocean is not included, as it has very little effect on the time scale of 1000 years or less. Working in the limit of infinite depth does not lead to an observable difference in the results.

Integrating $\psi(z,t)$ over the depth gives a characteristic length $\lambda(t)$, which can be interpreted as the extent of the penetration of the change from the surface

$$\lambda(t) = \int_0^\infty \psi(z,t)dz = \sqrt{\frac{A_z t}{\pi}} \exp\frac{-W^2 t}{4A_z} + \frac{Wt}{2} + \left(\frac{A_z}{W} + \frac{Wt}{2}\right) erf \frac{Wt}{2\sqrt{A_z t}} \tag{3}$$

The limiting value is $\lambda(\infty) = -A_z/W$, or 936 m using the numerical values of the constants.

In applications addressing the effects of temperature change at the ocean surface, the calculated average potential temperature change profile has to be added to the already existing steady-state thermocline profile. The approach to the limit as a function of depth and time after a step change is shown in Fig. 1.

The approach towards the new steady state can be examined by calculating the total change in the heat content of the oceans as a function of time. We integrate the temperature change profile over all depths and use 4186 J/(kg K) for the average specific heat and 3.61×10^8 km^2 for the global ocean surface area. The result is shown in Fig. 2, and the steady state limit is 1.41×10^{24} J of absorbed heat for a 1 K increase in the surface temperature.

The corresponding thermosteric increase in the global sea level can only be estimated approximately, as the thermal expansion coefficient varies significantly with temperature and salinity. Using the average value $\alpha = 1.7 \times 10^{-4}$/K the calculated change in sea level as a function of time is also shown in Fig. 2. The asymptotic value is 158 mm for each degree of surface warming.

3 Evaluation of the heat content and sea level change from surface temperatures

The measured variation in global sea surface temperatures may be considered as a series of small step changes. Exploiting the superposition property of Eq. (1) we may evaluate a composite change by adding a number of solutions for a step change, Eq. (2) multiplied by a change in average surface temperatures from year to year. If SST*(t)* is the externally given sea surface temperature, a series of small changes is multiplied by the step change solution Eq. (2) and integrated over time to obtain the change in temperature profile

$$\vartheta(z,t) = \int_0^t \frac{d\mathrm{SST}(t')}{dt'} \psi(z, t-t')dt' \tag{4}$$

The temperature change profile Eq. (4) was evaluated in discrete time. We use the step of one year corresponding to annual values of mean sea surface temperatures at the centre of the time step. Denoting the average sea surface temperature for the year i as SST*(i)* and using the solution for a step change, Eq. (2), the composite solution $\vartheta(z,n,k)$ describing the temperature change in year n resulting from the sea surface temperature changes beginning in the year k is evaluated as

$$\vartheta(z,n,k) = \sum_{i=k+1}^{n} [\mathrm{SST}(i) - \mathrm{SST}(i-1)]\psi(z, n-i+1). \tag{5}$$

We select the GISS average global ocean surface temperatures (GISS, 2009) shown in Fig. 3 as the input data in the evaluation. Details in the data may still need a correction for recording artifacts (Thompson et al., 2008), but major features are confirmed by many independent lines of evidence. Although temperatures in the GISS data set fluctuate from year to year we do not apply smoothing as the integration process itself provides sufficient averaging.

The warming of the global ocean calculated from the sea surface temperature data using Eq. (5) is illustrated in Fig. 4. In this and all the following figures it is important to notice that the effect of surface temperature changes before 1880 is not included in the calculation.

Integrating over finite depth ranges similar to the previous section leads to the corresponding changes in ocean heat content due to the surface temperature changes since 1880. The integration can still be performed analytically even though the resulting expressions are more complicated than the Eq. (3). The result for the upper 700 m is shown in Figs. 5 and 6 together with the estimate based on the comprehensive review and evaluation of all available in situ measurements (Domingues et al., 2008). The calculation has no adjustable parameters other than a single choice of the reference zero selected only once to make values in Fig. 5 approximately agree in the middle of the measurement period (years 1972–1978). The calculated heat content change for the depths 700–5000 m is still relatively small, as the surface change takes a long time to reach greater depths (see Fig. 1). Using the average value of the thermal expansion coefficient as in Fig. 2 leads to the estimated sea level change shown in Fig. 7.

4 Discussion

Use of the advection-diffusion model of Eq. (1) to calculate the average ocean heat content and associated changes in sea level is supported by the results shown in the previous section. The major evidence comes in the validation of the heat content 0–700 m calculated from the average sea surface temperature by the direct estimate available for the recent past (Domingues et al., 2008). The sea level change, expected to be less accurate, is also in very good agreement with the direct estimate for 0–700 m (Fig. 7). It is remarkable that the agreement is obtained without any adjustable parameters, using the values for eddy diffusivity and upward drift velocity determined by Munk (1966). It should also be noted that the diffusivity and drift velocity values were determined from data taken at larger depths, while we now show that they are also applicable in the

0–700 m range where most of the warming is observed.

The results presented earlier include only the effects driven by the surface warming since 1880. Estimates based on measurements (Domingues et al., 2008) show by far the largest contribution to thermosteric sea level change originating from upper 300 m, where the equilibration time scale is of the order of 50 years. Evaluation beginning in 1880 is hence adequate to describe most of the effect at that depth. However, the warming of the deep ocean is much slower, and earlier changes in the global climate may still be spreading to deeper layers. The sea level change originating from the ocean below 700 m evaluated here is small and we estimate about 0.05 mm/yr for the most recent 50 years. This value does not agree with 0.2±0.1 mm/yr conjectured by Domingues et al. (2008). The possibility of more distant past changes still being felt in the present day sea level adjustment needs to be further explored in order to improve the closure of the sea level change budget. Over the past two millennia, global temperature reconstruction (Jones and Mann, 2004) does not indicate changes comparable in magnitude to those of the recent past. The contribution to sea level change from the deep ocean warming is hence unlikely to be much larger than calculated and the major contribution to the rise is the ocean mass increase (Cazenave et al., 2009).

The method of calculating average ocean heat content and the associated sea level change using only surface temperatures opens up the possibility of expanding our knowledge into the past and future. As one such example, we show in Fig. 8 the projected rise in global thermosteric sea level for the hypothetical situation where all warming of the sea surface had stopped and the present average temperature is maintained into the future. The figure emphasizes the slow rate of the response. In earlier step change calculation, only about 70% of the total change happens in the first 200 years and this slow rate is also seen in the projection of Fig. 8. In this overly optimistic example we had 34 mm rise since 1880 and we are committed to a further 48 mm.

In summary, we show that a simple global average calculation of ocean warming based on the advection-diffusion model and using the values of the coefficients determined in the past from steady-state measurements accurately reproduces much more

detailed recent data. In the amount of input information required, there is a vast difference between the average and detailed calculations. The described method using global averages provides a useful tool to understand the past heat content and associated sea level changes and to project the effect of present-day surface temperature changes into the future.

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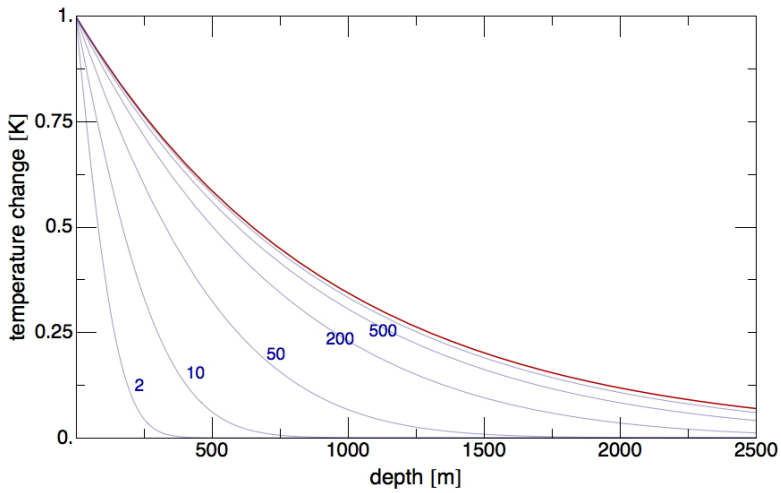

Fig. 1. Average potential temperature change profiles after a step change of temperature at the surface. The evaluation uses Munk's values for the vertical diffusivity and vertical drift velocity. The time after the change in years is shown in the figure for the first five curves. The top red curve is the steady-state limit and the one immediately below corresponds to 1000 years.

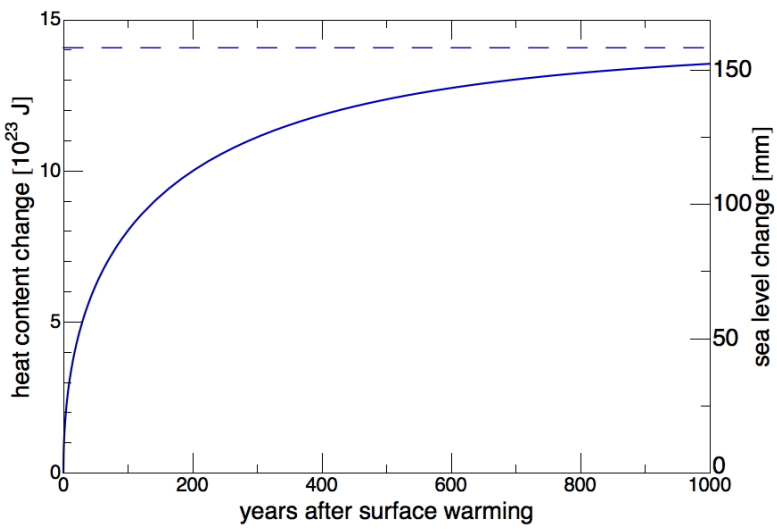

Fig. 2. Heat content change of the oceans following a 1°C step increase of the surface temperature. The right-hand scale shows the corresponding sea level change assuming an average thermal expansion coefficient of 1.7×10^{-4}/K.

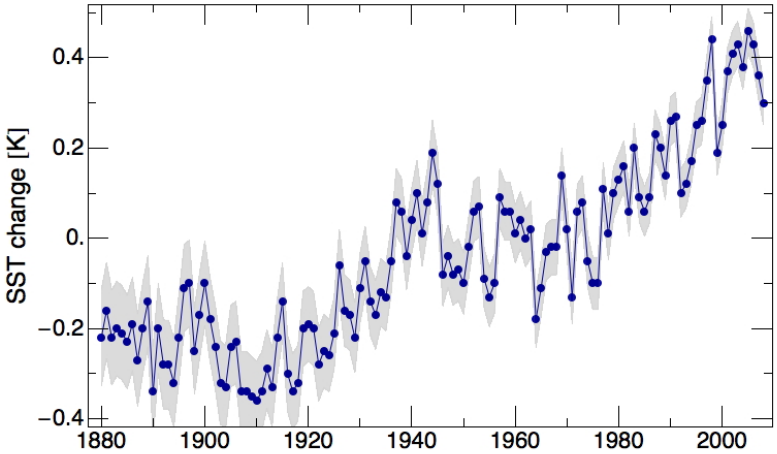

Fig. 3. NASA GISS ocean surface temperatures (GISS, 2009) used as the input in this study. The estimated 2σ error in the data (shown as shading) decreases from 0.1 K at the beginning of 20th century to 0.05 K in recent times (Hansen et al., 2006). In this and the subsequent figures, errors in the model results that originate from the errors in the input sea surface temperatures are not shown, because random errors mostly cancel in integration and systematic errors are not known.

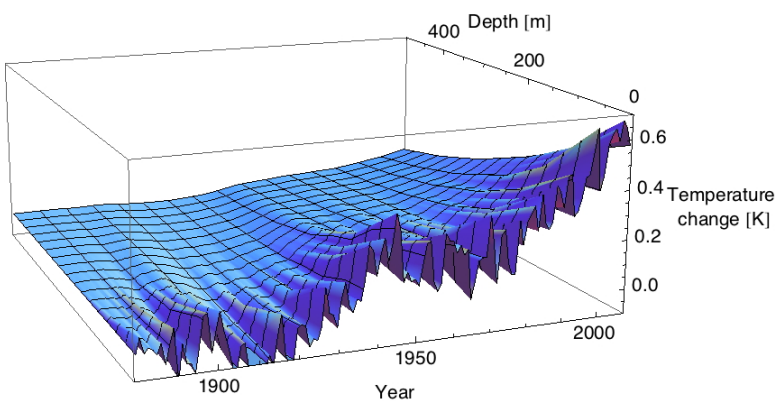

Fig. 4. Average temperature change in the top 500 m of the oceans resulting from surface temperature changes since 1880. At the deeper end of the figure annual variations are averaged out but decadal variations are still visible.

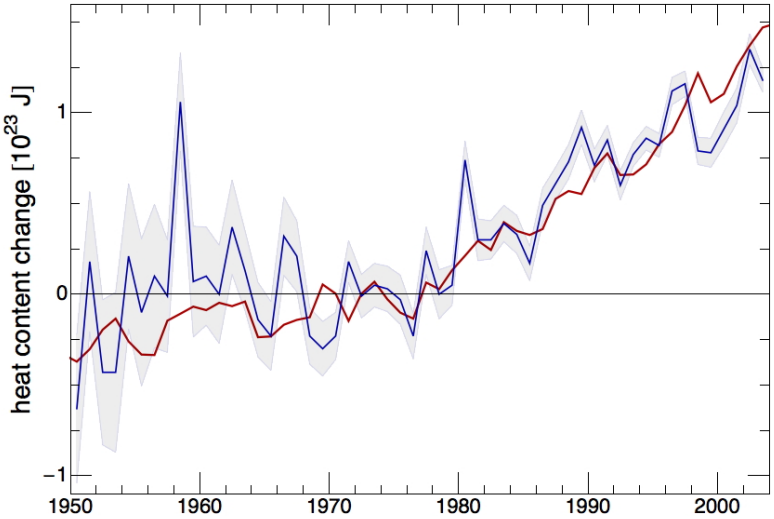

Fig. 5. Ocean heat content change 0–700 m calculated here from sea surface temperatures (red line) and estimated using in situ measurements (data by Domingues et al., 2008, blue line, shown together with one σ error). Both sets show annual values positioned in the middle of the year.

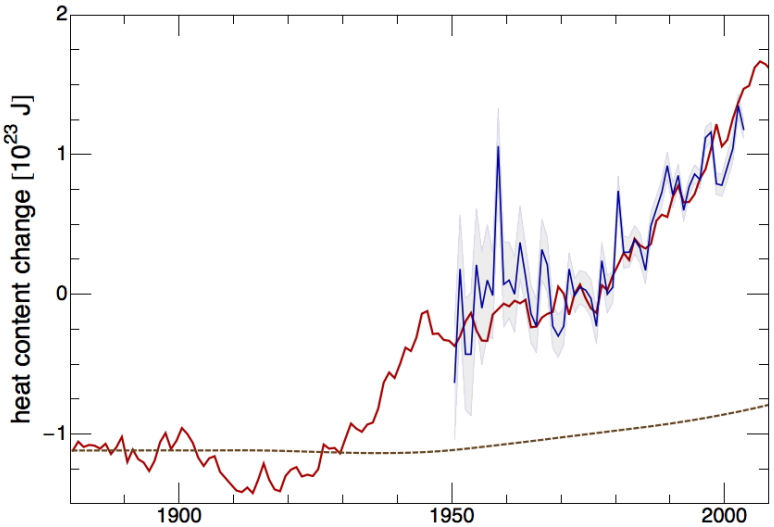

Fig. 6. Same as Fig. 5 but over the whole range of available sea surface temperatures for the recent past. The dashed line at the bottom shows the heat content change for depths larger than 700 m.

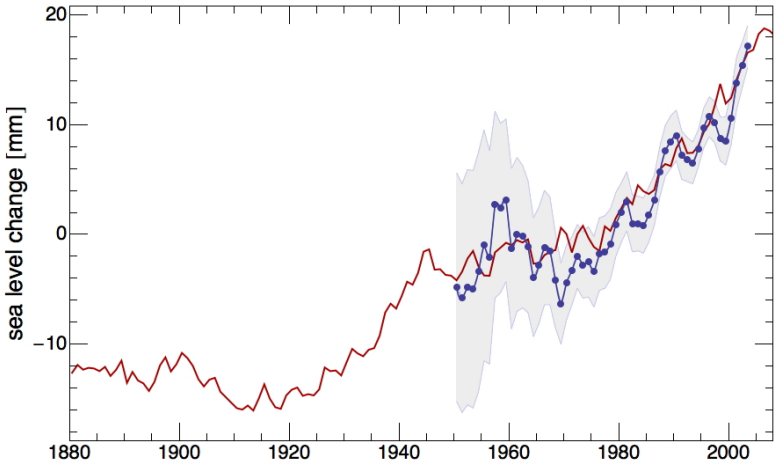

Fig. 7. Thermosteric sea level change resulting from the depths 0–700 m as calculated from annual sea surface temperatures (red line) and in situ measurements (Domingues et al., 2008, blue line, shown as a 3-year average with one σ error).

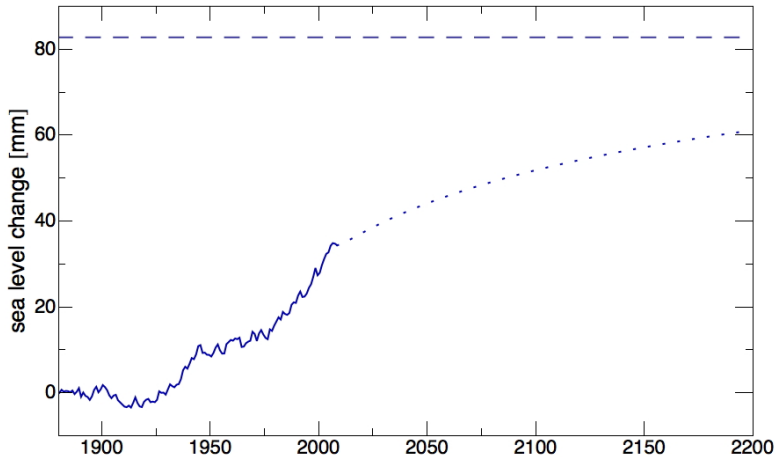

Fig. 8. Future projection of the thermosteric sea level rise based on the assumption that sea surface temperature will not increase beyond the changes of 1880–2008.